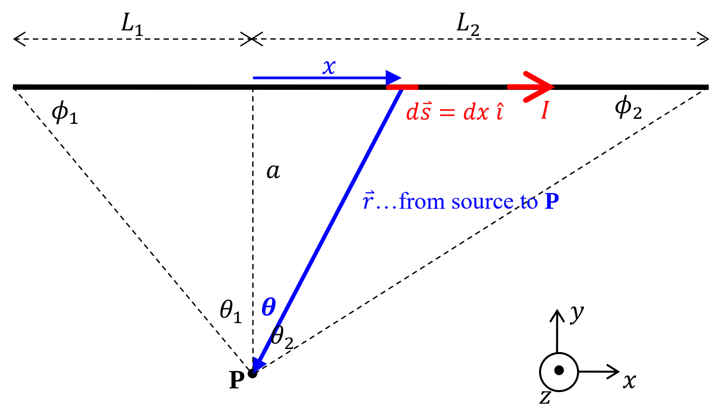
Chapter 29 Solutions

29.1 The Biot-Savart Law tells us



$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_i^f \frac{d\vec{s}\times\hat{r}}{r^2}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_i^f \frac{d\vec{s}\times\vec{r}}{r^3}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_i^f \frac{(dx\,\hat{\imath})\times(x(-\hat{\imath}) - a\hat{\jmath})}{(x^2+a^2)^{3/2}}$$

Do the cross-product before doing the integral.

$$\vec{B} = \frac{\mu_0 I}{4\pi}(-\hat{k})\int_i^f \frac{a\,dx}{(x^2+a^2)^{3/2}}$$

Form the picture at right, notice

$$\tan\boldsymbol{\theta} = \frac{x}{a}$$

$$x = a\tan\theta$$

$$dx = a\,d(\tan\theta)$$

$$dx = a\sec^2\theta\ d\theta$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}(-\hat{k})\int_i^f \frac{a\,(a\sec^2\theta\ d\theta)}{((a\tan\theta)^2+a^2)^{3/2}}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}(-\hat{k})\int_i^f \frac{a^2\ \sec^2\theta\ d\theta}{(a^2\tan^2\theta+a^2)^{3/2}}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}(-\hat{k})\int_i^f \frac{a^2\ \sec^2\theta\ d\theta}{(a^2)^{3/2}(\tan^2\theta+1)^{3/2}}$$

One of our favorite trig identities is

$$\sin^2\theta + \cos^2\theta = 1$$

Divide all terms by $\cos^2\theta$.

$$\tan^2\theta + 1 = \frac{1}{\cos^2\theta} = \sec^2\theta$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}(-\hat{k})\int_i^f \frac{a^2\ \sec^2\theta\ d\theta}{a^3(\sec^2\theta)^{3/2}}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi a}(-\hat{k})\int_i^f \frac{\sec^2\theta\ d\theta}{\sec^3\theta}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi a}(-\hat{k})\int_i^f \frac{d\theta}{\sec\theta}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi a}(-\hat{k})\int_i^f \cos\theta\,d\theta$$

$$\vec{B} = \frac{\mu_0 I}{4\pi a}(-\hat{k})\Big[\sin\theta\Big]_i^f$$

$$\vec{B} = \frac{\mu_0 I}{4\pi a}(-\hat{k})[\sin\theta_f - \sin\theta_i]$$

Problem continues on next page…

Usually the calculus steps can be skipped if you use the following memorized formula:

$$B_{straight\ segment} = \frac{\mu_0 I}{4\pi a}\left[\sin\theta_f - \sin\theta_i\right]$$

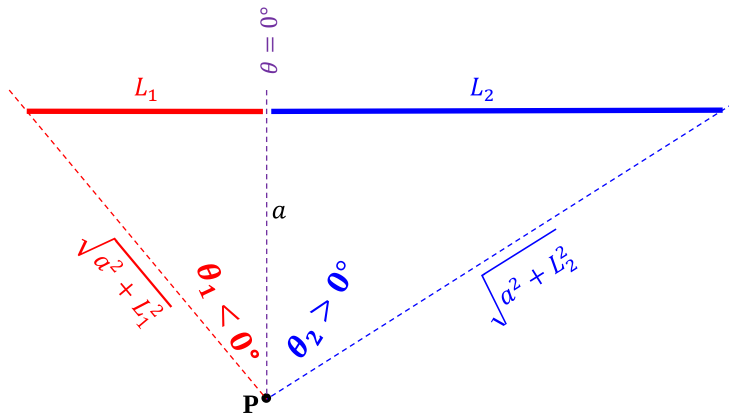
1) Here a is the distance from the point **P** to the segment along the $\perp$ bisector.
2) Use the angles of the segment endpoints from the $\perp$ bisector.
3) Tip: if you screw up the initial and final angles, no big deal. That simply switches the sign of the field. You can correct for this by checking field direction with the right hand rule.
4) WATCH OUT: angles to the *left* of the $\perp$ bisector are *negative*, while angles to the *right* of the $\perp$ bisector are *positive*. It is possible in some problems to have both angles positive or both angles negative.

The perpendicular bisector determines what angle is $\theta = 0°$. I chose to call angles to the right of the bisector positive.
From the picture I can see

$$\sin\theta_f = \sin\theta_2 = +\frac{L_2}{\sqrt{a^2 + L_2^2}}$$

$$\sin\theta_i = \sin\theta_1 = -\frac{L_1}{\sqrt{a^2 + L_1^2}}$$

Note: negative the angle makes $\sin\theta_1$ negative.



$$\vec{B} = \frac{\mu_0 I}{4\pi a}(-\hat{k})\left[\sin\theta_f - \sin\theta_i\right]$$

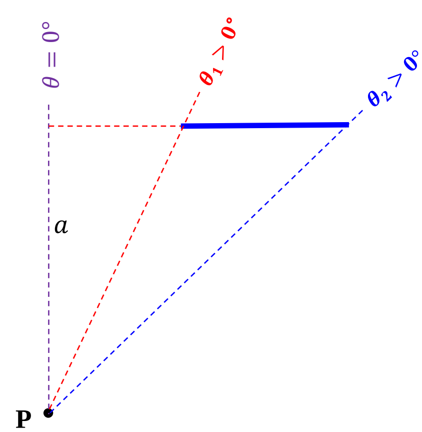
$$\vec{B} = \frac{\mu_0 I}{4\pi a}(-\hat{k})\left[\left(\frac{L_2}{\sqrt{a^2 + L_2^2}}\right) - \left(-\frac{L_1}{\sqrt{a^2 + L_1^2}}\right)\right]$$

$$\vec{B} = \frac{\mu_0 I}{4\pi a}(-\hat{k})\left[\left(\frac{L_2}{\sqrt{a^2 + L_2^2}}\right) + \left(\frac{L_1}{\sqrt{a^2 + L_1^2}}\right)\right]$$

Notice both terms contribute to the field positively. This makes sense, both wire segments produce magnetic field contributions in the same direction at P as determined by the right hand rule.

Every year someone asks:
"What happens if both angles are on the same side of the perpendicular bisector?"
Consider the figure shown at right.
The same exact process follows but now both angle as are positive

A final note on the angles: The angle is zero at the perpendicular bisector. You are free to choose the sign of angles to the right or left of this 0° angle *as long as you are consistent with signs*. For example, in the previous example I could've chosen to call both angles negative. What you should not do is call one positive and the other negative. Note: If you end up with a negative magnitude, just take the absolute value.



29.2 Spend some serious time digging into all the little nuggets in the figures at right.

I will show an alternate style that you may prefer on the next page….

The field created by the differential segment located on the *right* side of the loop is

$$d\vec{B}_1 = \frac{\mu_0 I}{4\pi} \cdot \frac{d\vec{s}_1 \times \vec{r}_1}{r_1^3}$$

The top view shows $d\vec{s}_1 = R\ d\theta\ (-\hat{\jmath})$.

Going *from the source* to the point of interest one finds

$$\vec{r}_1 = -R\hat{\imath} + z\hat{k}$$

Plugging into $d\vec{B}_1$

$$d\vec{B}_1 = \frac{\mu_0 I}{4\pi} \cdot \frac{R\ d\theta\ (-\hat{\jmath}) \times \left(-R\hat{\imath} + z\hat{k}\right)}{(R^2 + z^2)^{3/2}}$$

By looking at the symmetry in the figure, we expect only the $\hat{k}$ term should survive upon integration. We figure out which term(s) in the cross-product will *output* a $\hat{k}$ term. The only term which produces a $\hat{k}$ term comes from $\hat{\jmath} \times \hat{\imath}$. Eliminating all the rest gives

$$\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \int_0^{2\pi} \frac{R\ d\theta\ (-\hat{\jmath}) \times (-R\hat{\imath})}{(R^2 + z^2)^{3/2}}$$

Notice I switched from $d\vec{B}_1$ to $\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \int_0^{2\pi} blah\ blah\ blah$. I have to do this because my symmetry argument only holds after doing the entire integration!

$$\vec{B} = \frac{\mu_0 I}{4\pi}\left(-\hat{k}\right) \int_0^{2\pi} \frac{R^2\ d\theta}{(R^2 + z^2)^{3/2}}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\left(-\hat{k}\right) \frac{R^2\ (2\pi)}{(R^2 + z^2)^{3/2}}$$

$$\vec{B} = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}}\left(-\hat{k}\right)$$

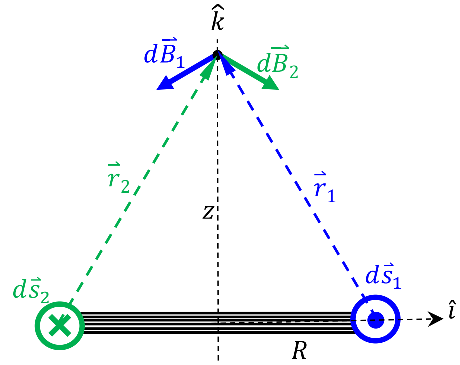
For N turns we find

$$\boldsymbol{\vec{B}} = \frac{\boldsymbol{\mu_0 N I R^2}}{\boldsymbol{2(R^2 + z^2)^{3/2}}}\left(-\boldsymbol{\hat{k}}\right)$$

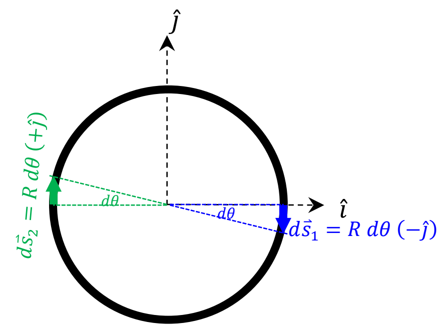
Note: the *magnitude* at the center ($z = 0$) of a single loop ($N = 1$) is

$$\boldsymbol{B_{center\ of\ single\ circular\ loop} = \frac{\mu_0 I}{2R}}$$

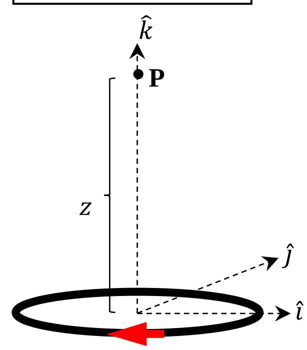
Pure Side View



Top View



Slanted Side View



The field created by an *arbitrary* differential segment is

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{d\vec{s} \times \vec{r}}{r^3}$$

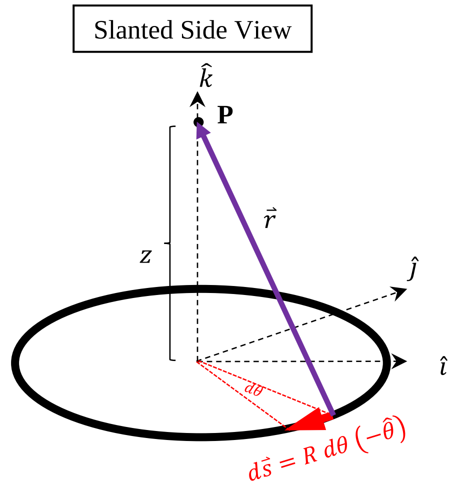
Notice $d\vec{s} = R\, d\theta\, \left(-\hat{\theta}\right)$.
Think: current flows *opposite the standard math direction* (counter-clockwise) for θ. Thus the $-\hat{\theta}$. Note: $\hat{\theta} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$.

Going *from the source* to the point of interest one finds

$$\vec{r} = R(-\hat{r}_{math}) + z\hat{k}$$

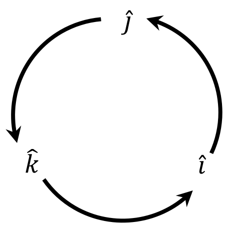
Here $-\hat{r}_{math}$ means a unit vector directed towards the center of the circle.

$$-\hat{r}_{math} = -(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$$

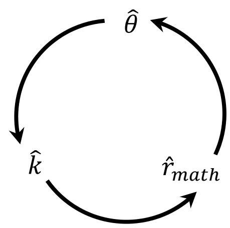


Tip: rather than converting everything to Cartesian, make a wheel of pain for cylindrical polar coordinates!

If



then



Plugging into

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{\left(R\, d\theta\, \left(-\hat{\theta}\right)\right) \times \left(R(-\hat{r}_{math}) + z\hat{k}\right)}{r^3}$$

By looking at the symmetry in the figure, we expect only the $\hat{k}$ term should survive upon integration.
We figure out which term(s) in the cross-product will *output* a $\hat{k}$ term.
The only term which produces a $\hat{k}$ term comes from $\left(-\hat{\theta}\right) \times (-\hat{r}_{math}) = -\hat{k}$.
Eliminating all the rest gives

$$\vec{B} = \frac{\mu_0 I}{4\pi}\left(-\hat{k}\right)\int_0^{2\pi} \frac{R^2\, d\theta}{(R^2 + z^2)^{3/2}}$$

$$\boldsymbol{\vec{B}} = \frac{\boldsymbol{\mu_0 N I R^2}}{\boldsymbol{2(R^2 + z^2)^{3/2}}}\left(-\boldsymbol{\hat{k}}\right)$$

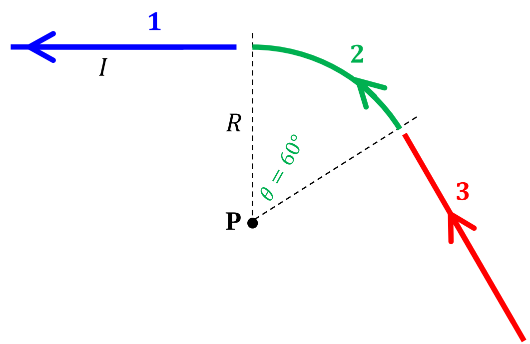
29.3 We know the total field is the *vector* sum of the fields produced by each segment.

$$\vec{B}_{total} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

In this case, each segment produces a contribution to the total field IN THE SAME DIRECTION (verify with right hand rule).
When this occurs, we may simplify the above expression and *add the field magnitudes instead*.

$$B_{tot} = B_1 + B_2 + B_3$$

Notice, from symmetry in the problem, $B_1 = B_3$.
These wires are each exactly half of an infinite wire.



To be clear, a *half-infinite* wire extends from infinity to the PERPENDICULAR BISECTOR. More on this below the final answer…

Together they produce a field magnitude equal to that of a single infinitely long wire ($B_1 + B_3 = \frac{\mu_0 I}{2\pi R} = 0.1592\frac{\mu_0 I}{R}$)!

$$B_2 = \frac{1}{6}\ of\ the\ field\ magntiude\ produced\ by\ a\ full\ circle\ at\ center$$

$$B_2 = \frac{1}{6}\cdot\frac{\mu_0 I}{2R} = \frac{1}{12}\cdot\frac{\mu_0 I}{R} = 0.08333\frac{\mu_0 I}{R}$$

$$B_{tot} = 0.1592\frac{\mu_0 I}{R} + 0.08333\frac{\mu_0 I}{R}$$

$$\boldsymbol{B_{tot} = 0.242\frac{\mu_0 I}{R}}$$

b) The magnetic field created by the wire is directed *out of the page* at **P**.

Regarding a *half-infinite* wire…

To see where this comes from, consider our shortcut formula

$$B = \frac{\mu_0 I}{4\pi a}\left[\sin\theta_f - \sin\theta_i\right]$$

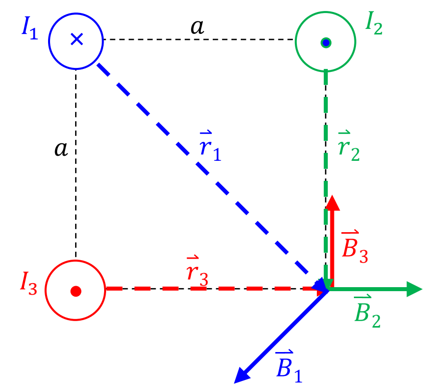
The angles you punch in for θ_f & θ_i correspond to the enpoints of the wire segment you are working on.
Be sure to determine the angles *relative to the perpendicular bisector*!
The perpendicular bisector corresponds to $\theta = 0$.
If both angles are on the same side of the bisector, make both angles positive.
If angles are on opposite sides of the bisector, make one of them negative.
Since I am looking for a field magnitude in the above equation, I expect a positive result.
I could always take the absolute value of the output to ensure a positive result.
Alternatively, simply make sure your largest angle is positive and use it for θ_f.
For a half-infinite wire, one angle is $\theta_f = 90°$ corresponding to the end of the wire segment near infinity.
The other angle is $\theta_i = 0°$ corresponding to the end of the wire segment at the perpendicular bisector.

29.4 The *magnitude* produced by each long wire is given by the memorized result for long straight wires distance r away:

$$B = \frac{\mu_o i}{2\pi r}$$

To get the directions, first draw a vector from source to point of interest for each wire. The direction of magnetic field produced at the point of interest is always $\perp$ to this source to POI vector.

- GRAB THAT WIRE (align your right hand thumb in the direction of current).
- Curl your fingers around the wire until your hand is aligned with the source to POI $\vec{r}$
- The direction your fingertips are curling indicates the direction of the magnetic field at the POI!



$$\vec{B}_{total} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

$$\vec{B}_{total} = \frac{\mu_o I_1}{2\pi r_1}\left(\cos 45°\,(-\hat{\imath}) + \sin 45°\,(-\hat{\jmath})\right) + \frac{\mu_o I_2}{2\pi r_2}(+\hat{\imath}) + \frac{\mu_o I_3}{2\pi r_3}(+\hat{\jmath})$$

$$\vec{B}_{total} = -\frac{\mu_o I_1}{2\pi r_1}\left(\frac{\sqrt{2}}{2}\hat{\imath} + \frac{\sqrt{2}}{2}\hat{\jmath}\right) + \frac{\mu_o I_2}{2\pi r_2}(+\hat{\imath}) + \frac{\mu_o I_3}{2\pi r_3}(+\hat{\jmath})$$

$$\vec{B}_{total} = -\frac{\sqrt{2}}{2}\cdot\frac{\mu_o I_1}{2\pi r_1}(\hat{\imath} + \hat{\jmath}) + \frac{\mu_o I_2}{2\pi r_2}(+\hat{\imath}) + \frac{\mu_o I_3}{2\pi r_3}(+\hat{\jmath})$$

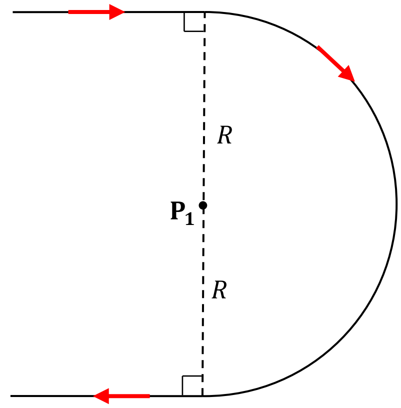
In the problem statement we are told $I_2 = I_3 = I$ & $I_1 = 2I$.
From the figure we can see $r_1 = \sqrt{2}a$ & $r_2 = r_3 = a$.

$$\vec{B}_{total} = -\frac{\sqrt{2}}{2}\cdot\frac{\mu_o(2I)}{2\pi\left(\sqrt{2}a\right)}(\hat{\imath} + \hat{\jmath}) + \frac{\mu_o(I)}{2\pi(a)}(+\hat{\imath}) + \frac{\mu_o(I)}{2\pi(a)}(+\hat{\jmath})$$

$$\vec{B}_{total} = ZERO$$

Nothing like a long, painful calculation that ends up being ZERO! Physicist attempt at humor?

29.5



a) For the big half circle (the left figure) current must run to the right in the upper straight wire segment to cause a magnetic field into the page $\mathbf{P_1}$.
For the small half circle (the right figure) current must run upwards in the upper straight wire segment to cause a magnetic field into the page at $\mathbf{P_2}$.
b) The *left* wire can be thought of as two "half-infinite" straight wire plus half a circle of radius R.

$$\vec{B}_1 = 2(half\ an\ infinite\ wire)(-\hat{k}) + \frac{1}{2}(circlular\ wire\ at\ center)(-\hat{k})$$

In this case, all fields contribute in the same direction to the net field. When this happens we can drop the vectors and add the magnitudes (not always the case).

$$B_1 = 2(half\ an\ infinite\ wire) + \frac{1}{2}(circlular\ wire\ at\ center)$$

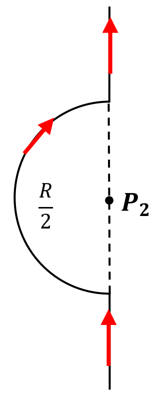
$$B_1 = (infinite\ wire) + \frac{1}{2}(circlular\ wire\ at\ center)$$

$$B_1 = \frac{\mu_0 I_1}{2\pi R} + \frac{1}{2}\left(\frac{\mu_0 I_1}{2R}\right)$$

$$B_1 = \frac{\mu_0 I_1}{R}\left(\frac{1}{2\pi} + \frac{1}{4}\right)$$

$$B_1 = \frac{\mu_0 I_1}{R}(0.1592 + 0.25)$$

$$B_1 = 0.4092\frac{\mu_0 I_1}{R}$$



Notice the straight wire contributes slightly less than the half circle.
The *right* wire has a half circle segment (of radius $\frac{R}{2}$) and two straight line segments.
Because the straight line segments run *radially* towards or away from $\mathbf{P_2}$ they do not contribute to the magnetic field at $\mathbf{P_2}$.
Obviously they contribute to the magnetic field produced at many *other* locations, they simply don't contribute to the field at $\mathbf{P_2}$.

$$B_2 = \frac{1}{2}(circlular\ wire\ at\ center) = \frac{1}{2}\left(\frac{\mu_0 I_2}{2\left(\frac{R}{2}\right)}\right) = 0.5\frac{\mu_0 I_2}{R}$$

Notice the field at $\mathbf{P_2}$ is stronger than the field at $\mathbf{P_1}$ **IF** the *same* current is used.
The half circle segment of wire 2 is twice as close to the point of interest; that contribution is twice as large!!!

Question asked for ratio of currents to make mag field at the two points identical in size.
This means

$$0.4092\frac{\mu_0 I_1}{R} = 0.5\frac{\mu_0 I_2}{R}$$

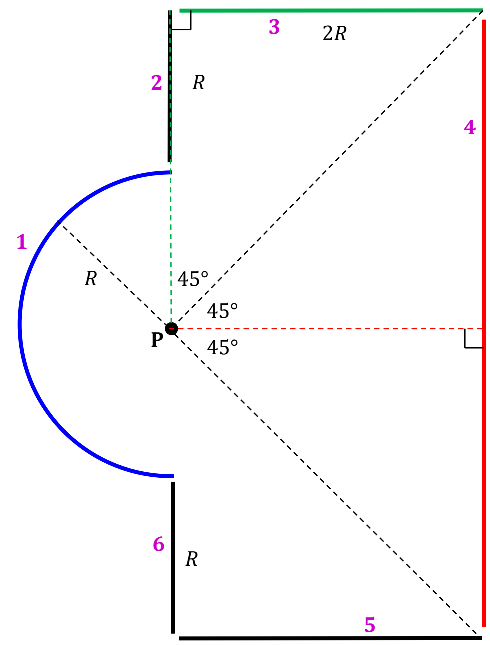
$$\frac{I_1}{I_2} = \frac{0.5}{0.4092} = 1.222$$

Current in case 1 must be about 22% larger to cause the same size field at the center of its half circle.

29.6 **Note:** for N turns, the current simply gets multiplied by N!

a) Current runs counter-clockwise in the loop.

b) I tried to split up the segments so they would be slightly easier to identify. Note: there are many other correct ways to split up the loop as we will see throughout the course of doing this problem.



Segments **2** & **6** will not contribute to the field at **P** because in those segments current runs either radially away or towards **P**.

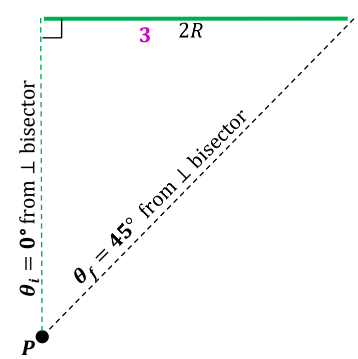
All other segments produce contributions to the magnetic field in the same direction. Once again we can ignore the vectors and simply add field magnitude from each segment. This will not always be the case.

By symmetry, we see the magnetic field contribution from **3** & **5** should be the same. Furthermore, look carefully and realize segment **4** should be exactly twice as large as either segment **3** or **5**.

Note: for straight wire segments I have my handy memorized formula:

$$B_{straight\ segment} = \frac{\mu_0 NI}{4\pi a}\left[\sin\theta_f - \sin\theta_i\right]$$

Here a is the distance from the point **P** to the segment along the ⊥ bisector.
Use the angles of the segment endpoints from the ⊥ bisector.
Tip: if you screw up the initial and final angles, no big deal. That simply switches the sign of the field. You can correct for this by checking field direction with the right hand rule.
WATCH OUT: angles to the *left* of the ⊥ bisector are *negative*, while angles to the *right* of the ⊥ bisector are *positive*. It is possible in some problems to have both angles positive or both angles negative.



I will compute the field contribution from segment 3.

$$B_3 = \frac{\mu_0 NI}{4\pi(\mathbf{2R})}[\sin(\mathbf{45°}) - \sin(\mathbf{0°})]$$

$$B_3 = \frac{\mu_0 NI}{8\pi R}\left[\frac{\sqrt{2}}{2} - 0\right]$$

$$B_3 = \frac{\sqrt{2}\mu_0 NI}{16\pi R} = 0.02814\frac{\mu_0 NI}{R}$$

Finally, the field contribution from segment **1** is easy to compute: $B_1 = \frac{1}{2}\left(\frac{\mu_0 NI}{2R}\right) = 0.25\frac{\mu_0 NI}{R}$.

The total field is thus

$$\vec{B}_{tot} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5 + \vec{B}_6$$

Since all contributions point in the same direction we can add magnitudes instead (not always the case).
Segments **2** & **6** do not contribute. By symmetry $\vec{B}_3 = \vec{B}_5$ and $\vec{B}_4 = 2\vec{B}_3$.

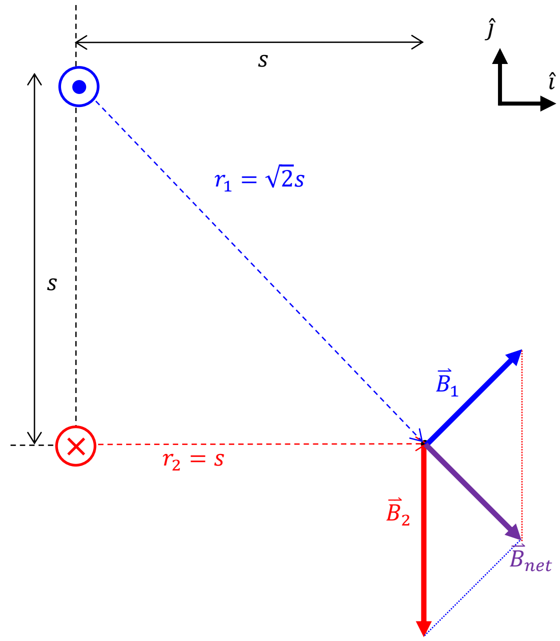
$$B_{tot} = B_1 + 4B_3 = 0.3626\frac{\mu_0 NI}{R}$$

$$\boldsymbol{I} = \frac{\mathbf{2.76}\boldsymbol{RB}}{\boldsymbol{N\mu_0}}$$

c) The largest field contribution comes from segment 1, followed by 4, then 3 & 5 are tied. Finally 2 & 6 are tied.

29.7

a) Fortunately, this problem uses square symmetry so angles will be 45° or 90°. See the picture at right.



$$\vec{B}_{net} = \vec{B}_1 + \vec{B}_2$$

$$\vec{B}_{net} = \frac{\mu_0 I}{2\pi(\sqrt{2}s)}(\cos 45° \,(\hat{\imath}) + \sin 45° \,(\hat{\jmath})) + \frac{\mu_0 I}{2\pi(s)}(-\hat{\jmath})$$

$$\vec{B}_{net} = \frac{\mu_0 I}{2\pi(\sqrt{2}s)}\left(\frac{\sqrt{2}}{2}(\hat{\imath} + \hat{\jmath})\right) - \frac{\mu_0 I}{2\pi s}\hat{\jmath}$$

$$\vec{B}_{net} = \frac{\mu_0 I}{4\pi s}(\hat{\imath} + \hat{\jmath}) - \frac{\mu_0 I}{2\pi s}\hat{\jmath}$$

$$\vec{B}_{net} = \frac{\mu_0 I}{4\pi s}(\hat{\imath} - \hat{\jmath})$$

$$B_{net} = \sqrt{\left(\frac{\mu_0 I}{4\pi s}\right)^2 + \left(-\frac{\mu_0 I}{4\pi s}\right)^2}$$

$$B_{net} = \frac{\mu_0 I}{4\pi s}\sqrt{2}$$

From problem statement we are given $B_{net} = B$ and side length s. Asks us to solve for I.

$$\boldsymbol{I = \frac{4\pi s B}{\sqrt{2}\mu_0}}$$

b) In the figure one sees the direction is 45° below the positive x-axis.

29.8

a) $a\sqrt{2} = \frac{r}{2}$ gives $r = 2\sqrt{2}a$

b) into the page

c) segments 2 and 4 (notice they point radially away/towards the point of interest)

d) segments 1 and 5 ARE NOT SEMI-INFINITE WIRES!!! One way is to view segments 1 and 5 as an infinite wire MINUS the little bit in the middle or determine segment 1 (or 5) and double it. I choose to do segment 5 and double it.

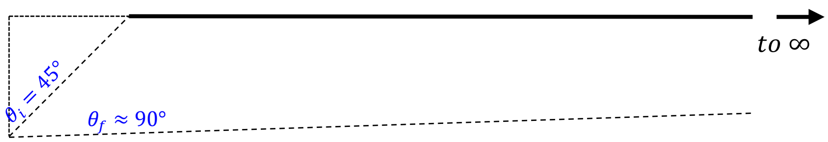
$$\vec{B}_5 = \frac{\mu_0 I}{4\pi}\int_i^f \frac{(dx\hat{\imath}) \times (-x\hat{\imath} - a\hat{\jmath})}{(x^2 + a^2)^{3/2}}$$

$$\vec{B}_5 = \frac{\mu_0 I a}{4\pi}(-\hat{k})\int_i^f \frac{dx}{(x^2 + a^2)^{3/2}}$$

$$\vec{B}_5 = \frac{\mu_0 I a}{4\pi}(-\hat{k})\left[\frac{\sin\theta}{a^2}\right]_i^f$$

$$\vec{B}_5 = \frac{\mu_0 I}{4\pi a}(-\hat{k})[\sin\theta_f - \sin\theta_i]$$

In this case remember that all angles must be referenced to the perpendicular bisector!



Therefore we find

$$\vec{B}_5 = \frac{\mu_0 I}{4\pi a}(-\hat{k})[\sin 90° - \sin 45°] = \frac{\mu_0 I}{4\pi a}\left(1 - \frac{\sqrt{2}}{2}\right)(-\hat{k})$$

Don't forget $\vec{B}_1 = \vec{B}_5$.

Lastly $B_3 = \frac{1}{4}B_{circle} = \frac{1}{4}\frac{\mu_0 I}{2r}$. Watch out…the radius of the circle is r…not a!!! From part a recall $r = 2\sqrt{2}a$.

Also, by some right hand rule one sees $\vec{B}_3$ points in the same direction as $\vec{B}_1$ & $\vec{B}_5$.

This allows us to add the magnitudes of the field.

Putting it all together and plugging in all numbers to a calculator

$$B_{total} = B_3 + 2B_5 = \boldsymbol{0.0908\frac{\mu_0 I}{a}}$$

29.9

a) In each case the circular arc closest to the point (smallest radius) will be the dominant contributor to the magnetic field. Using the right hand rule tells us the directions of the mag fields at each black dot are

CASE 1	CASE 2	CASE 3
Into the page $(-\hat{k})$	Out of the page $(+\hat{k})$	Into the page $(-\hat{k})$

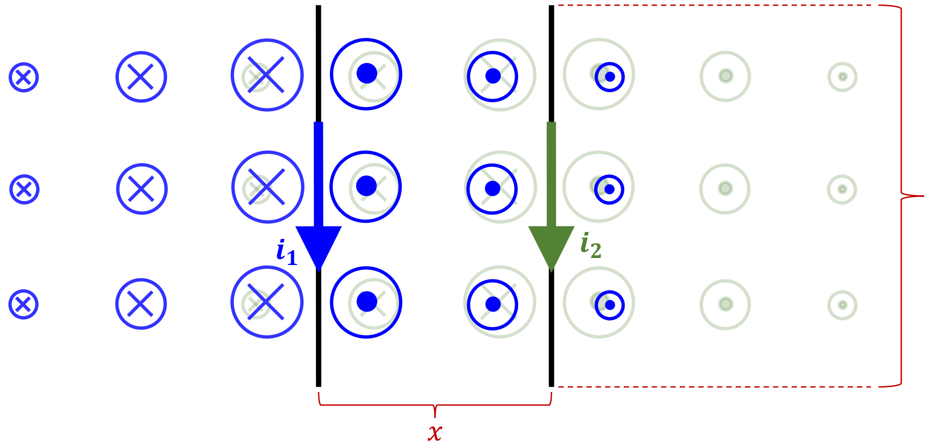
b) In each case the straight line segments carry current either radially towards or away from the black dot. The straight line segments thus do not contribute to the mag field at the black dot. Furthermore, in each case we can add (or subtract) the circular segments to determine the total mag field. The magnetic field of a full circle is $\frac{\mu_0 I}{2r}$. See work below. I'm assuming $\hat{\imath}$ to the right, $\hat{\jmath}$ up, and $\hat{k}$ out of the page.

CASE 1	**CASE 2**	**CASE 3**
$\vec{B}_{net} = \frac{3}{4}\left(\frac{\mu_0 I}{2(2r)}\hat{k} - \frac{\mu_0 I}{2(r)}\hat{k}\right)$	$\vec{B}_{net} = \frac{1}{2}\left(\frac{\mu_0 I}{2(3r)}\hat{k} + \frac{\mu_0 I}{2(r)}\hat{k}\right)$	$\vec{B}_{net} = \frac{1}{2}\left(-\frac{\mu_0 I}{2(r)}\hat{k} + \frac{\mu_0 I}{2(3r)}\hat{k}\right)$
$\vec{B}_{net} = \frac{\mu_0 I}{r}\left(\frac{3}{4}\right)\left(\frac{1}{4} - \frac{1}{2}\right)\hat{k}$	$\vec{B}_{net} = \frac{\mu_0 I}{r}\left(\frac{1}{2}\right)\left(\frac{1}{6} + \frac{1}{2}\right)\hat{k}$	$\vec{B}_{net} = \frac{\mu_0 I}{r}\left(\frac{1}{2}\right)\left(-\frac{1}{2} + \frac{1}{6}\right)\hat{k}$
$\vec{B}_{net} = \frac{\mu_0 I}{r}\left(\frac{3}{4}\right)\left(-\frac{1}{4}\right)\hat{k}$	$\vec{B}_{net} = \frac{\mu_0 I}{r}\left(\frac{1}{2}\right)\left(\frac{4}{6}\right)\hat{k}$	$\vec{B}_{net} = \frac{\mu_0 I}{r}\left(\frac{1}{2}\right)\left(-\frac{2}{6}\right)\hat{k}$
$\vec{B}_{net} = \frac{\mu_0 I}{r}\left(\mathbf{\frac{3}{16}}\right)(-\hat{k})$	$\vec{B}_{net} = \frac{\mu_0 I}{r}\left(\mathbf{\frac{1}{3}}\right)(\hat{k})$	$\vec{B}_{net} = \frac{\mu_0 I}{r}\left(\mathbf{\frac{1}{6}}\right)(-\hat{k})$

b) Notice the *magnitudes* of the fields are now easily ranked by the colored fractions.
Converting to decimals makes it even easier to rank the field magnitudes.

$$B_3 < B_1 < B_2$$

29.10 Part a) I actually ended up drawing the field created by wire 1 in blue and the field created by wire 2 in green. To reduce clutter, I made the green lines mostly transparent.



Part b) The force exerted by mag field 1 on wire 2 is directed TO THE LEFT.

Part c) As long as $x \ll L$, we may assume our wires are infinitely long.
Assuming this is true, current 1 creates a *field* at wire 2's position (distance x from wire 1)

$$B_1 = \frac{\mu_0 i_1}{2\pi x}$$

The *force* exerted by wire 1 on wire 2 is thus

$$F_{1on2} = i_2 L_2 B_1$$

$$F_{1on2} = i_2 L_2 \left(\frac{\mu_0 i_1}{2\pi x}\right)$$

$$\boldsymbol{F_{1on2} = \frac{\mu_0 i_1 i_2 L}{2\pi x}}$$

It's common to express this result as *force per unit length* (exerted by wire 1 on wire 2):

$$\frac{F_{1on2}}{L} = \frac{\mu_0 i_1 i_2}{2\pi x}$$

Note: if x is large, usually the fields are too weak to cause any significant force anyways.
Again, this problem usually only matters in real-world applications when $x \ll L$.

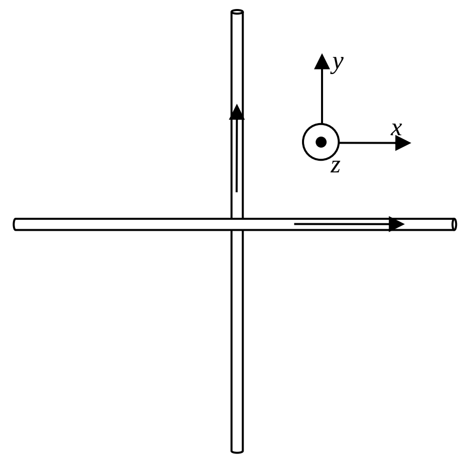
Part d) Doubling either current doubles the size of the force.
Reversing either current direction reverses the direction of the force.

Part e) Like *currents* attract; like *charges* repel. [Video of wires connected to car battery (no additional resistance).]
An interesting way to look at it: consider the two magnetic field patterns produce above.
If the wires are attracted to each other (and could somehow overlap), notice the field patterns would be identical.
This is analogous to bar magnets trying to line up their magnetic fields by snapping north pole to south pole!

Part f) 3.65 mN (to the right on wire 1, to the left on wire 2).
Note: technically the question asked for force (not force magnitude).
As such you should specify both magnitude and direction.

Curious about high power transmission lines? I started googling around and found some extremely boring technical leaflets and websites. One stated the smallest spacing they ever used was about $x \approx 8$ ft ≈ 2.4 m.
Another site stated the metal supports are perhaps $L = 700$ ft ≈ 210 m apart.
Another website stated typical transmission currents average less than 1000 A.
I punched in some numbers and found a tiny force of about 20 N on that massive cable.
THINK: the weight of a 700 ft long cable (or wind forces from a slight breeze) easily dwarf this magnetic force.

29.11 By symmetry one expects the net force to be zero. Consider the force on the horizontal wire due to the current in the vertical wire. The field will point into the page on one side and out of the page on the other side. Even though the B-field gets weaker, it will weaken symmetrically on each side. On the right half of the wire a next force will point upwards while on the left half a net force will point downwards. Notice that while there is no net force, there <u>is</u> a net <u>torque</u> that would tend to align the two wires! This makes me the case of long parallel wires running current in the same direction that were attracted to each other…



To get the force on the *left half* of the horizontal wire:

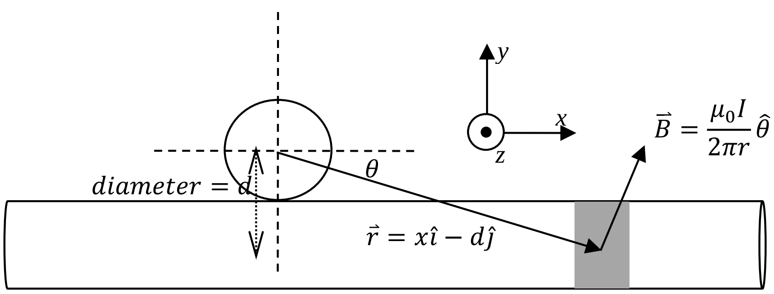
$$\vec{F}_{left\ half} = \int_0^{L/2} I d\vec{s} \times \vec{B}_{ext}$$

Here $\vec{B}_{ext}$ is caused by the *vertically* oriented wire while we are integrating over the length of the *horizontal* wire. In the end we can consider infinitely long wires by letting $L \to \infty$. <u>One might INCORRECTLY think the following is reasonable:</u>

$$\vec{F}_{left\ half} = I \int_0^{L/2} dx\hat{\imath} \times \frac{\mu_0 I}{2\pi x}\left(-\hat{k}\right) = \frac{\mu_0 I^2}{2\pi}\hat{\jmath}\int_0^{L/2}\frac{dx}{x} = \frac{\mu_0 I^2}{2\pi}\hat{\jmath}\ln\frac{L/2}{0} = UNDEFINED!$$

Of course, here is where the mathematical model fails a reality check. It would seem that the point where the two wires touch would contribute an infinite amount of force. In reality, the two wires would probably be covered with some insulator of some sort. The wires could not be infinitely thin. As such, at the point where the two wires touch the horizontal wire is actually displaced in the positive z-direction ever so slightly. In doing this, at the point where the two wires touch the magnetic field is actually parallel to the horizontal wire and contributes absolutely no force! Very counterintuitive!

To solve this, I would view the system from the top and create a polar coordinate system (see figure below…note I used this coordinate system so we could still use the standard definition of $\hat{\theta}$). You need to include an angle as well as the diameter of the wires in your calculation. Also, it would still be only an approximation…



$$\vec{F}_{left\ half} = \frac{\mu_0 I^2}{2\pi}\int_0^{L/2}\frac{dx\left(\hat{\imath}\times\hat{\theta}\right)}{(x^2+d^2)^{1/2}}$$

which still needs work before integrating since $\hat{\theta} = -\sin\theta\hat{\imath} + \cos\theta\hat{\jmath} = -\frac{d}{(x^2+d^2)^{1/2}}\hat{\imath} + \frac{x}{(x^2+d^2)^{1/2}}\hat{\jmath}$

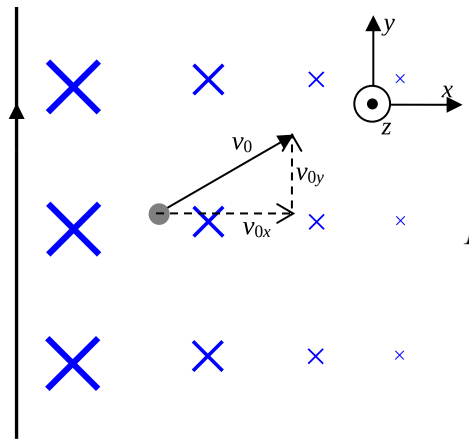
Don't forget…this is only approximate! How brutally delicious!

29.12

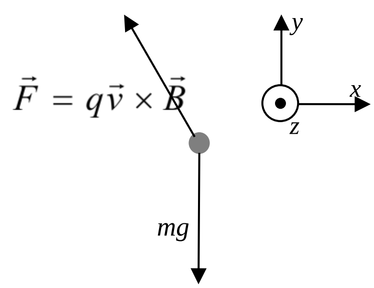
a) Mag field created by wire points into the page at location of ball. For a ball falling down, we expect downward velocity. RHR gives force on ball to the right. HOWEVER, at the instant after the ball is released velocity is essentially zero so one wouldn't expect much deflection at all.

b) Instead of being released from rest, the charge was thrown with initial speed v_0. If the charge is thrown into (or out of) the page ($\pm\hat{k}$) there is no initial magnetic force.

c)



FBD



$$\vec{v} = v_0 \cos\theta\,\hat{\imath} + v_0 \sin\theta\,\hat{\imath}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi x}(-\hat{k})$$

Could do $\vec{F_B} = q\vec{v} \times \vec{B}$

Or recognize $F_B = qv_0 B \sin 90$ in a direction that is 90° from v_0. Either way you should end up with:

$$\vec{a} = -\frac{qv_0 B}{m}\sin\theta\,\hat{\imath} + \left(\frac{qv_0 B}{m}\cos\theta - g\right)\hat{\jmath}$$

$$a = \sqrt{\left(\frac{qv_0 B}{m}\right)^2 + g^2 - \frac{2gqv_0 B}{m}\cos\theta}$$

where $B = \frac{\mu_0 I}{2\pi x}$. **Nasty.**

d) What would the path of travel would look like? The magnetic field will not be constant unless the particle is moving vertically. Unfortunately, even when the particle moves vertically for an instant, it will be deflected from vertical and not experience a constant field. As such, this particle will definitely not have a constant force (even if gravity is negligible). To solve this one would need to derive the position as a function of time. To do this, one would need some serious diff eqt'n skills. This is because you find:

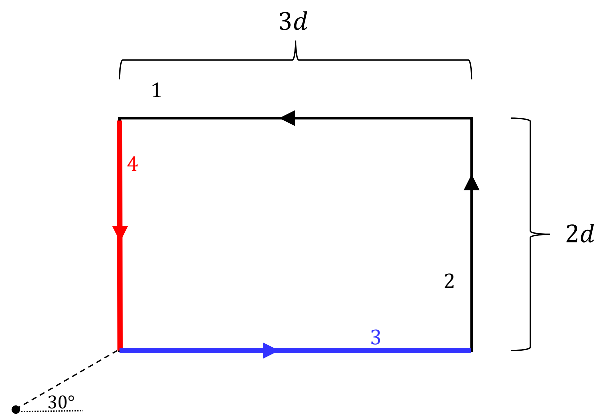
$$\vec{a} = \frac{d^2(\vec{x})}{dt^2} = -\frac{qv_0 B}{m}\sin\theta\,\hat{\imath} + \left(\frac{qv_0 B}{m}\cos\theta - g\right)\hat{\jmath}$$

where both B and θ depend on x. I don't how this would work out off the top of my head. I think there might even be various wildly different answers based on the field strength and the initial speed…I have no clue! <u>Create me a time machine and I'll work on this when I am really old and come back to give you the answers.</u> Then again, clearly you didn't get the time machine done…thanks for nothing.

<u>**Best method: code it.**</u>

29.13

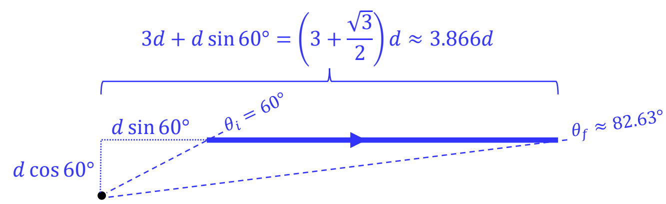
a) The segments farthest from point **P** should contribute the least, segments 1 & 2 in this case.



b) Segments 3 & 4 both produce contributions INTO the page at **P**. Since these are the dominant contributions, the *net* field points into the page at **P**.

c) My guess: I think the contribution from segment 4 should be slightly larger than the contribution from segment 3…but I bet it is very close.

d) I sketched the angles for segment 3 in blue. Notice I had to extend the wire to find the perpendicular bisector.

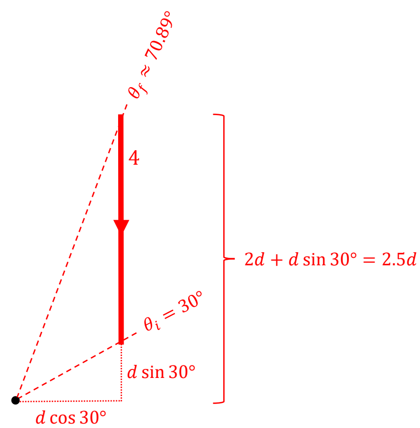


$$\theta_f = \tan^{-1}\left(\frac{3d + d\sin 60°}{d\cos 60°}\right) \approx 82.63°$$

$$B_3 \approx \frac{\mu_0 I}{4\pi(d\cos 60°)}[\sin 82.63° - \sin 60°]$$

$$B_3 \approx 0.02001\frac{\mu_0 I}{d}$$

I sketched the angles for segment 4 in red. Notice I had to extend the wire to find the perpendicular bisector. Notice the perpendicular distance is not the same!



$$\theta_f = \tan^{-1}\left(\frac{2d + d\sin 30°}{d\cos 30°}\right) \approx 70.89°$$

$$B_4 \approx \frac{\mu_0 I}{4\pi(d\cos 30°)}[\sin 70.89° - \sin 30°]$$

$$B_4 \approx 0.04088\frac{\mu_0 I}{d}$$

Evidently it wasn't that close…

Solution continues on the next page…

e) I expected the other segments to be negligible…to be sure I compute their contributions.
The net field from segments 3 & 4 is

$$\vec{B}_{34} \approx -0.06089 \frac{\mu_0 I}{d} \hat{k}$$

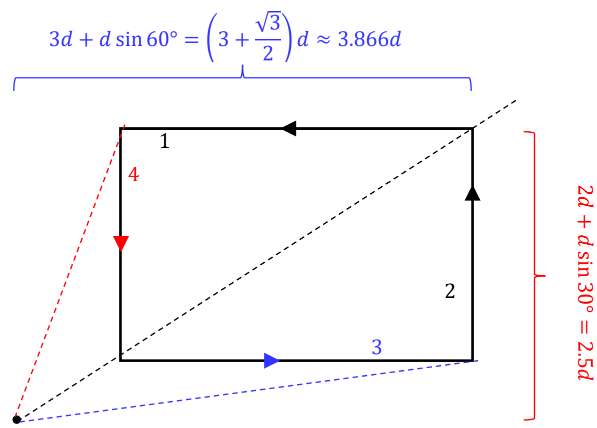
where I'm assuming a standard coordinate system with out of the page equivalent to $\hat{k}$.
Why am I suddenly writing the field vector (instead of field magnitude)?
Because the contributions from segments 1 & 2 point the opposite direction!
When **P** is *inside* a loop, each segment produces a field contribution in the *same* direction.
When **P** is *outside* a loop, *opposite* segments produce field contributions in the *opposite* directions.

For segment 1 you can show

$$\theta_f = \tan^{-1}\left(\frac{3d + d\sin 60°}{2d + d\sin 30°}\right) \approx 57.11°$$

$$B_1 \approx \frac{\mu_0 I}{4\pi(2d + d\sin 30°)}[\sin 57.11° - \sin(19.11°)]$$

$$\vec{B}_1 \approx 0.01631 \frac{\mu_0 I}{d} \hat{k}$$



For segment 2 you can show

$$\theta_f = \tan^{-1}\left(\frac{2d + d\sin 30°}{3d + d\sin 60°}\right) \approx 32.89°$$

$$B_2 \approx \frac{\mu_0 I}{4\pi(3d + d\sin 60°)}[\sin 32.89° - \sin(7.37°)]$$

$$\vec{B}_2 \approx 0.00854 \frac{\mu_0 I}{d} \hat{k}$$

The combined field from segments 1 & 2 is

$$\vec{B}_{12} \approx +0.02485 \frac{\mu_0 I}{d} \hat{k}$$

The percent change to the field MAGNITUDE B_{34} is

$$\%\Delta B = \frac{\Delta B_{34}}{B_{34}} \times 100\%$$

$$\%\Delta B = \frac{-B_{12}}{B_{34}} \times 100\%$$

$$\%\Delta B = \frac{-0.00854 \frac{\mu_0 I}{d}}{0.06089 \frac{\mu_0 I}{d}} \times 100\%$$

$$\%\boldsymbol{\Delta B} \approx \mathbf{-41}\%$$

Clearly the other segments should NOT be considered negligible.

29.14

a) See the derivation in problem 29.1. The field at **P** is

$$B = \frac{\mu_0 I}{4\pi d}\left[\left(\frac{\frac{L}{2}}{\sqrt{d^2 + \frac{L^2}{4}}}\right) + \left(\frac{\frac{L}{2}}{\sqrt{d^2 + \frac{L^2}{4}}}\right)\right]$$

$$\boldsymbol{B = \frac{\mu_0 I L}{4\pi d\sqrt{d^2 + \frac{L^2}{4}}}}$$

b) We are asked to find a value for L such that

$$B = 0.99 B_{\infty\, straight}$$

$$\frac{\mu_0 I L}{4\pi d\sqrt{d^2 + \frac{L^2}{4}}} = 0.99\frac{\mu_0 I}{2\pi d}$$

$$\frac{L}{2\sqrt{d^2 + \frac{L^2}{4}}} = 0.99$$

$$\frac{L}{2} = 0.99\sqrt{d^2 + \frac{L^2}{4}}$$

Now square both sides.

$$\frac{L^2}{4} = 0.9801\left(d^2 + \frac{L^2}{4}\right)$$

$$\frac{1}{4(0.9801)}L^2 = d^2 + \frac{L^2}{4}$$

$$\frac{1}{0.9801}L^2 = 4d^2 + L^2$$

$$\left(\frac{1}{0.9801} - 1\right)L^2 = 4d^2$$

$$L^2 = \frac{4}{\frac{1}{0.9801} - 1}d^2$$

$$L = \sqrt{\frac{4}{\frac{1}{0.9801} - 1}}\,d$$

$$\boldsymbol{L \approx 14.04d}$$

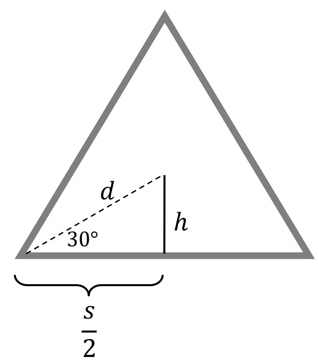
Another interpretation of this:

Suppose you are distance d from a long straight wire.

Points on the wire more than $7d$ from the perpendicular bisector produce negligible contributions to the field (compared to contributions from points on the wire near the perpendicular bisector).

29.15 $\vec{B}$ on axis through centroid of equilateral triangle

TOP VIEW

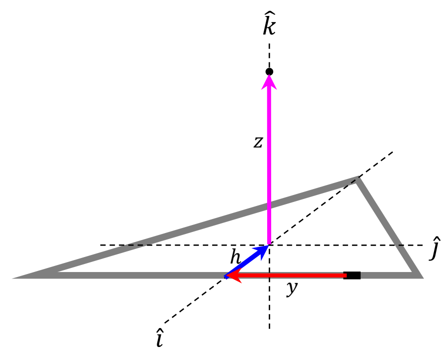


First consider the top view shown in the upper figure at right. I will call the distance from the center of an edge to the centroid h. From a corner to the centroid will be d. Note: because this is an equilateral triangle, we know the angle 30° shown in the figure. Using SOH CAH TOA we find

$$\tan 30° = \frac{h}{s/2}$$

$$h = \frac{s \tan 30°}{2} = \frac{s}{2\sqrt{3}}$$

SIDE VIEW



Now consider the triangular loop of wire carrying current I shown in the side view (lower figure at right). We are going to use the Biot-Savart law to compute the magnetic field at some point on the z-axis. The loop lies in the xy-plane. The z-axis runs through the centroid of the triangle. The point of interest is distance z above the xy-plane.

We will start by using the tiny segment of wire indicated by the little black rectangle distance y from the x-axis. The vector from the segment to the point of interest is thus

$$\vec{r} = -h\hat{\imath} - y\hat{\jmath} + z\hat{k}$$

The distance is thus

$$r = \sqrt{h^2 + y^2 + z^2}$$

If current runs $+\hat{\jmath}$ in this segment we expect $d\vec{s} = dy\hat{\jmath}$.
Using the Biot-Savart law <u>on this segment</u> gives

$$\vec{B}_{seg\ 1} = \frac{\mu_0 I}{4\pi} \int_i^f \frac{d\vec{s} \times \hat{r}}{r^2}$$

$$\vec{B}_{seg\ 1} = \frac{\mu_0 I}{4\pi} \int_i^f \frac{d\vec{s} \times \vec{r}}{r^3}$$

$$\vec{B}_{seg\ 1} = \frac{\mu_0 I}{4\pi} \int_{-s/2}^{+s/2} \frac{(dy\hat{\jmath}) \times \left(-h\hat{\imath} - y\hat{\jmath} + z\hat{k}\right)}{(h^2 + y^2 + z^2)^{3/2}}$$

Notice the $\hat{\jmath} \times \hat{\jmath}$ terms will drop. If we consider the symmetry of all three segments of the wire, we expect the final result should be entirely in the $\hat{k}$. Using this symmetry we can ignore the $\hat{\jmath} \times \hat{k}$ term as well!!! Therefore

$$\vec{B}_{seg\ 1z} = \frac{\mu_0 I}{4\pi} \int_{-s/2}^{+s/2} \frac{(dy\hat{\jmath}) \times (-h\hat{\imath})}{(h^2 + y^2 + z^2)^{3/2}}$$

$$\vec{B}_{seg\ 1z} = \frac{\mu_0 I}{4\pi} (\hat{\jmath}) \times (-\hat{\imath}) \int_{-s/2}^{+s/2} \frac{h\, dy}{(h^2 + y^2 + z^2)^{3/2}}$$

$$\vec{B}_{seg\ 1z} = \frac{\mu_0 I}{4\pi} \hat{k} \int_{-s/2}^{+s/2} \frac{h\, dy}{(h^2 + y^2 + z^2)^{3/2}}$$

It is worth checking the expected direction with a right hand rule. I confirmed it should be $+\hat{k}$. Lastly, we know there are three segments which produce equal contribution in the $+\hat{k}$ direction. The total field produced is thus

$$\vec{B} = 3\vec{B}_{seg\ 1z} = \frac{3\mu_0 I}{4\pi} \hat{k} \int_{-s/2}^{+s/2} \frac{h\, dy}{(h^2 + y^2 + z^2)^{3/2}}$$

From here the integral is doable but a bit ugly.
More on the next page…

First note that both h and z are constants for the integral. To clean things up I choose to let

$$\alpha^2 = h^2 + z^2$$

Also, because the integral is an *even* function with *symmetric* limits we can cut the limits in half and multiply by two. We now have

$$\vec{B} = \frac{6\mu_0 I h}{4\pi}\hat{k}\int_0^{s/2} \frac{dy}{(y^2+\alpha^2)^{3/2}}$$

$$\vec{B} = \frac{3\mu_0 I h}{2\pi}\hat{k}\int_0^{s/2} \frac{dy}{(y^2+\alpha^2)^{3/2}}$$

An integral table gives me

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}$$

This gives

$$\vec{B} = \frac{3\mu_0 I h}{2\pi}\hat{k}\left[\frac{y}{\alpha^2\sqrt{y^2+\alpha^2}}\right]_0^{s/2}$$

$$\vec{B} = \frac{3\mu_0 I h}{2\pi}\frac{s/2}{\alpha^2\sqrt{\frac{s^2}{4}+\alpha^2}}\hat{k}$$

Now some the ugly part. Recall from the very beginning $h = \frac{s}{2\sqrt{3}}$. This gives $\alpha^2 = \frac{s^2}{12} + z^2$. Plugging in this garbage gives

$$\vec{B} = \frac{3\mu_0 I\left(\frac{s}{2\sqrt{3}}\right)}{2\pi}\frac{s/2}{\left(\frac{s^2}{12}+z^2\right)\sqrt{\frac{s^2}{4}+\left(\frac{s^2}{12}+z^2\right)}}\hat{k}$$

Cleaning up all that crap gives the final result

$$\vec{B} = \frac{\sqrt{3}\mu_0 I}{8\pi}\frac{s^2}{\left(\frac{s^2}{12}+z^2\right)\sqrt{\frac{s^2}{3}+z^2}}\hat{k}$$

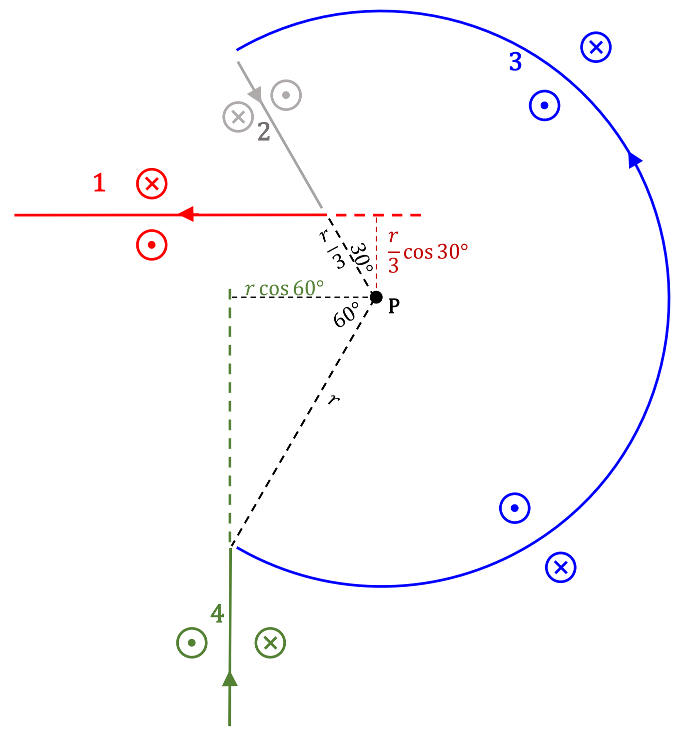
We have already checked the direction. Is there another way to check this result? I can check in several ways.

1) Always check the units. We expect results for $\vec{B}$ should have $\mu_0 I$ on top and meters on bottom. Check!
2) Consider the result if $z = 0$. This is a much simpler problem. You should be able to do this one for an exam. I did this check and the results agreed. $\vec{B}_{z=0} = \frac{9}{2\pi}\frac{\mu_0 I}{s}$.
3) For *large* values of z ($z \gg s$) the magnetic field should be given by $\vec{B} \approx \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}$ where $\vec{\mu} = I\vec{A}$ is the magnetic moment of the loop. I did this check and the results agreed. $\vec{B}_{z\gg s} = \frac{\sqrt{3}}{8\pi}\frac{\mu_0 I s^2}{z^3}$.
4) I was able to dig around and find the result for a *square* loop of wire. This result was

$$\vec{B}_{square} = \frac{\mu_0 I}{2\pi}\frac{s^2}{\left(\frac{s^2}{4}+z^2\right)\sqrt{\frac{s^2}{2}+z^2}}\hat{k}$$

I notice this result has a similar form.

29.15½ It is easier to label large figures…make yours large. Segments are labeled in the figure shown at right.



SEGMENT 2

Segment **2** produces no contribution since current in segment **2** runs radially towards the point of interest.

SEGMENT 3

From the original figure, we know the circular arc comprises $\frac{240°}{360°} = \frac{2}{3}$ of an entire circle. Direction from right hand rule.

$$\vec{B}_3 = \frac{2}{3} \cdot \frac{\mu_0 I}{2\pi r} \hat{k} \approx 0.10610 \frac{\mu_0 I}{r} \hat{k}$$

SEGMENT 1

First I determine the perpendicular bisector distance as

$$a = \frac{r}{3} \cos 30° = \frac{r}{2\sqrt{3}}$$

Now use

$$B_1 = \frac{\mu_0 I}{4\pi \left(\frac{r}{2\sqrt{3}}\right)} \left(\sin \theta_f - \sin \theta_i\right)$$

Since both angles θ_i & θ_f lie on the same side of the perpendicular bisector they should use the same sign. Since I want field magnitude, I should make both angles negative (or absolute value the result to get a magnitude).

$$B_1 = \frac{\mu_0 I}{4\pi \left(\frac{r}{2\sqrt{3}}\right)} [\sin(-60°) - \sin(-90°)]$$

$$\vec{B}_1 = \frac{\sqrt{3}\mu_0 I}{4\pi r} \hat{k} \approx 0.13783 \frac{\mu_0 I}{r} \hat{k} \qquad \text{direction from right hand rule}$$

SEGMENT 4

First I determine the perpendicular bisector distance as

$$a = r \cos 60° = \frac{r}{2}$$

Now use

$$B_4 = \frac{\mu_0 I}{4\pi \left(\frac{r}{2}\right)} \left(\sin \theta_f - \sin \theta_i\right)$$

Since both angles θ_i & θ_f lie on the same side of the perpendicular bisector they should use the same sign. Since I want field magnitude, I should make both angles negative (or absolute value the result to get a magnitude).

$$B_4 = \frac{\mu_0 I}{4\pi \left(\frac{r}{2}\right)} [\sin(-30°) - \sin(-90°)]$$

$$\vec{B}_4 = \left(2 - \sqrt{3}\right) \frac{\mu_0 I}{4\pi r} (-\hat{k}) \approx 0.02132 \frac{\mu_0 I}{r} (-\hat{k}) \qquad \text{direction from right hand rule}$$

Recall we were told $\vec{B}_{total} = B\hat{k}$. From this one finds

$$B\hat{k} = \vec{B}_1 + \vec{B}_3 + \vec{B}_4 \approx 0.2226 \frac{\mu_0 I}{r} \hat{k}$$

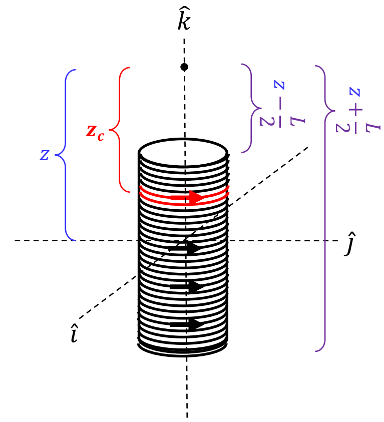
$$\boldsymbol{I \approx 4.49 \frac{r}{\mu_0 B}}$$

29.16

Consider a small slice of the solenoid as a coil.
The general result for the field created by a coil is

$$\vec{B}_{coil} = \frac{\mu_0 N_c I r^2}{2(r^2 + z_c^2)^{3/2}}\hat{k}$$

Here z_c is distance from the center of the *coil* (along the axis of the coil).
Watch out! Here N_c is the number of turns in the coil (not the solenoid).



In the problem statement, we defined z as the distance from the center of the *solenoid*.
We also stated the solenoid had N turns.
Please notice the distinctions between z & z_c and N & N_c.

$$\vec{B}_{solenoid} = \textit{the sum of all the small } \vec{B}_{coil} \textit{ contributions}$$

$$\vec{B}_{solenoid} = \int_i^f d\vec{B}_{coil}$$

$$\vec{B}_{solenoid} = \int_i^f \frac{\mu_0 N_c I r^2}{2(r^2 + z_c^2)^{3/2}}\hat{k}$$

Perhaps you are wondering where the differential is hiding?
Consider the number of turns in the coil.
Our particular coil is some tiny thickness dz_c.
We know the solenoid uses $n = \frac{N}{L}$ turns per unit length.
Therefore

$$N_c = n\, dz_c$$

$$\vec{B}_{solenoid} = \int_{z-\frac{L}{2}}^{z+\frac{L}{2}} \frac{\mu_0 n I r^2 dz_c}{2(r^2 + z_c^2)^{3/2}}\hat{k}$$

$$\vec{B}_{solenoid} = \frac{\mu_0 n I r^2 \hat{k}}{2}\int_{z-\frac{L}{2}}^{z+\frac{L}{2}} \frac{dz_c}{(r^2 + z_c^2)^{3/2}}$$

From the integral table one finds

$$\vec{B}_{solenoid} = \frac{\mu_0 n I \boldsymbol{r^2} \hat{k}}{2}\left(\frac{z_c}{\boldsymbol{r^2}\sqrt{r^2 + z_c^2}}\right)_{z-\frac{L}{2}}^{z+\frac{L}{2}}$$

Notice the bold $\boldsymbol{r^2}$ in the numerator out front and the denominator inside the parentheses will cancel!

$$\vec{B}_{solenoid} = \frac{\mu_0 n I}{2}\left(\frac{z + \frac{L}{2}}{\sqrt{r^2 + \left(z + \frac{L}{2}\right)^2}} - \frac{z - \frac{L}{2}}{\sqrt{r^2 + \left(z - \frac{L}{2}\right)^2}}\right)\hat{k}$$

WATCH OUT! This equation was derived using

$$\vec{B}_{coil} = \frac{\mu_0 N_c I r^2}{2(r^2 + z_c^2)^{3/2}}\hat{k}$$

This formula for the coil is only valid *along the axis of the coil*!
As such, our solenoid formula is only valid along the axis of the solenoid.

29.17

In part a we are asked to rank the magnitudes of the net external field at wire A caused by the other two wires. First get the direction of ach contribution using a right hand rule:

1. Point your right thumb in direction of current
2. Fingers or right hand curl around the wire in the direction of $\vec{B}$).

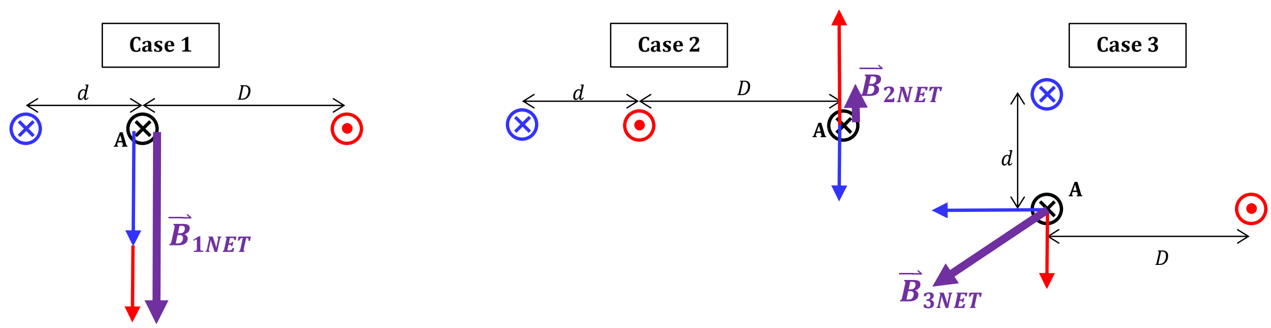
Note: recall the field *magnitude* for an INFINITE straight wire is

$$B_{\infty\ straight\ wire} = \frac{\mu_o I}{2\pi r}$$

This implies wires farther away produce smaller field contributions.

My pictures below show the NET field at each wire **A** in **purple.**

$$\boldsymbol{B_1 > B_3 > B_2}$$



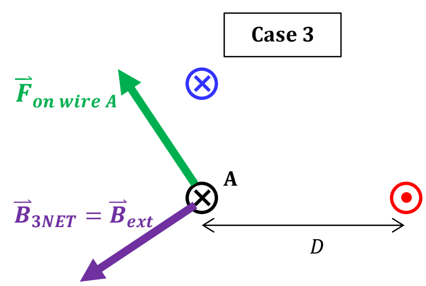
In part b we determine direction of force on wire **A** caused by the net field produced by the other two wires. Since all wires are nearly infinite, $\vec{B}_{ext}$ to wire **A** has uniform magnitude & direction at every point along wire **A**. When this is true:

$$\vec{F}_{on\ wire\ \mathbf{A}} = I_{wire\ \mathbf{A}} \vec{L}_{wire\ \mathbf{A}} \times \vec{B}_{ext}$$

To get the *direction* of this cross-product, use the following right hand rule:

1. Align the fingers of your right hand in the direction of the current $I_{wire\ \mathbf{A}}$.
2. Curl your fingers to the direction of the external field $\vec{B}_{ext}$.
3. Thumb points in the direction of the force.

$\vec{F}_{on\ wire\ \mathbf{A}}$ points up & to the left, slightly closer to the *vertical* axis.



29.18

a) The loops each carry the same current.
The field produced by a coil has magnitude

$$B_{coil} = \frac{\mu_0 N I r^2}{2(r^2 + z^2)^{3/2}}$$

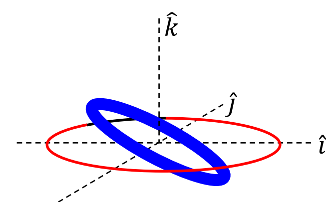
At the origin we may set $\boldsymbol{z = 0}$ giving

$$B_{coil} = \frac{\mu_0 I}{2r}$$

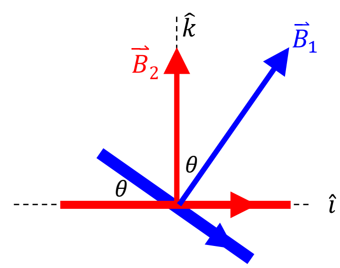
Here I assumed each loop has $N = 1$ unless otherwise specified.
The *smaller* loop produces a *larger* field vector at the origin.
It is impossible for these two field vectors to completely cancel each other because they have different magnitudes!

b) Using the figures at right I found

Slanted side view



Pure side view



$$\vec{B}_1 = \frac{\mu_0 I}{2a_1}(\sin\theta\,\hat{\imath} + \cos\theta\,\hat{k})$$

$$\vec{B}_2 = \frac{\mu_0 I}{2a_2}(\hat{k})$$

$$\vec{B}_{NET} = \vec{B}_1 + \vec{B}_2$$

$$\vec{B}_{NET} = \frac{\mu_0 I}{2}\left\{\frac{1}{a_1}\sin\theta\,\hat{\imath} + \left(\frac{1}{a_1}\cos\theta + \frac{1}{a_2}\right)\hat{k}\right\}$$

Note: in part c we were asked to plot field MAGNITUDE!

To keep it all on the same page in the sol'ns, I'll get magnitude now.

$$B_{NET} = \frac{\mu_0 I}{2}\sqrt{\left(\frac{1}{a_1}\sin\theta\right)^2 + \left(\frac{1}{a_1}\cos\theta + \frac{1}{a_2}\right)^2}$$

$$B_{NET} = \frac{\mu_0 I}{2}\sqrt{\left(\frac{1}{a_1}\sin\theta\right)^2 + \left(\frac{1}{a_1}\cos\theta\right)^2 + \left(\frac{1}{a_2}\right)^2 + \frac{2}{a_1 a_2}\cos\theta}$$

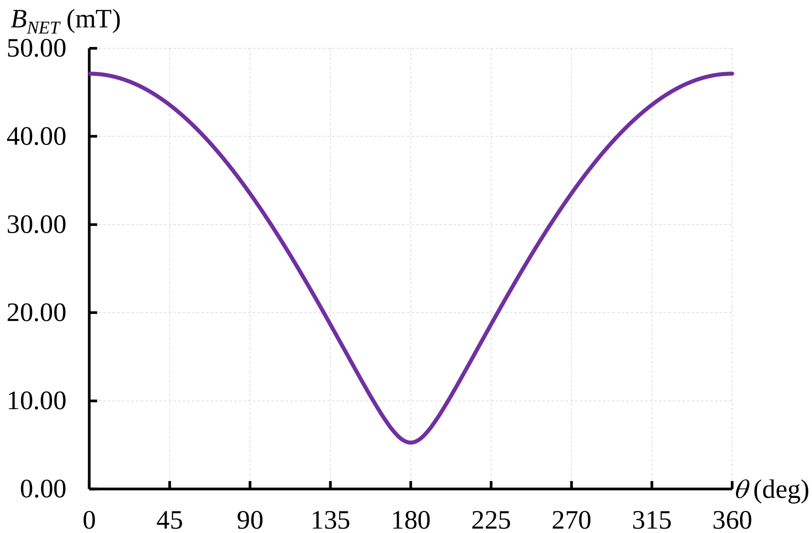
$$B_{NET} = \frac{\mu_0 I}{2}\sqrt{\frac{1}{a_1^2} + \frac{1}{a_2^2} + \frac{2}{a_1 a_2}\cos\theta}$$

$$B_{NET} = \frac{\left(4\pi \times 10^{-7}\,\frac{\mathrm{T\cdot m}}{\mathrm{A}}\right)(5.00\ \mathrm{A})}{2}\sqrt{\frac{1}{(0.12\ \mathrm{m})^2} + \frac{1}{(0.15\ \mathrm{m})^2} + \frac{2}{(0.12\ \mathrm{m})(0.15\ \mathrm{m})}\cos\theta}$$

$$\boldsymbol{B_{NET} = \left(3.142 \times 10^{-6}\sqrt{113.9 + 111.1\cos\theta}\right)\ \textbf{in units of T}}$$

Solution continues on the next page…

c) We are asked to make a plot of field magnitude versus angle.
Notice the magnitude of the net field never drops completely to zero.
The field magnitude does reach a minimum at 180°. This makes sense.
When the smaller loop is inverted the two field directions are in opposition.



d) We are asked which angles cause maximum/minimum torque on the loop.
I find the following equation is helpful when considering torques on loops.

$$\vec{\tau} = \vec{\mu} \times \vec{B}_{ext}$$

Recall $\vec{\mu}$ points in the direction of the area vector of the loop.
No torque occurs when $\vec{\mu}$ (from loop 1) & $\vec{B}_{ext}$ (caused by loop 2) are *parallel* (or *anti-parallel*)!
We expect *zero* torque when $\theta = 0° \ \ \& \ \ 180°$.

We expect torque (magnitude) is largest when $\vec{B}_{ext}$ (caused by loop 2) is *perpendicular* to $\vec{\mu}$.
We expect torque (magnitude) is largest when $\vec{B}_{ext}$ (caused by loop 2) is *in the plane of the loop*.
We expect *maximum* torque (magnitude) when $\theta = 90° \ \ \& \ \ 270°$.

29.19

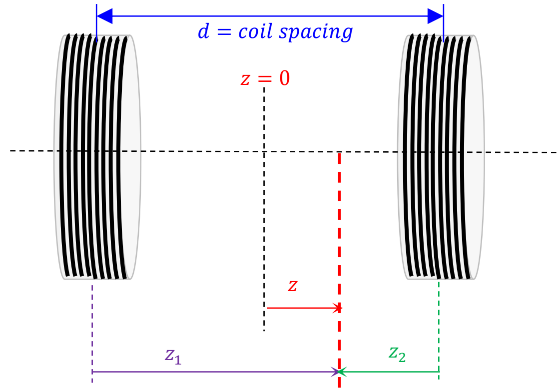
a) Use the superposition principle to sum the fields created by each ring.

$$\vec{B}_{total} = \vec{B}_1 + \vec{B}_2$$

Recall the magnetic field (magnitude) produced by a current-carrying coil is

$$B_{coil} = \frac{\mu_0 N I r^2}{2(r^2 + z)^{3/2}}$$

In this formula, z is the distance from the center of the coil. To add the fields of our two coils together, we must consider using $z_1 = z + \frac{d}{2}$ & $z_2 = z - \frac{d}{2}$ as shown in the figure at right.



$$\vec{B}_{total} = \frac{\mu_0 N I r^2}{2(r^2 + \boldsymbol{z_1^2})^{3/2}}\hat{k} + \frac{\mu_0 N I r^2}{2(r^2 + \boldsymbol{z_2^2})^{3/2}}\hat{k}$$

$$\vec{B}_{total} = \frac{\mu_0 N I r^2 \hat{k}}{2}\left\{\frac{1}{(r^2 + \boldsymbol{z_1^2})^{3/2}} + \frac{1}{(r^2 + \boldsymbol{z_2^2})^{3/2}}\right\}$$

$$\vec{B}_{total} = \frac{\mu_0 N I r^2 \hat{k}}{2}\left\{\frac{1}{\left(r^2 + \left(\frac{d}{2} + \boldsymbol{z}\right)^2\right)^{3/2}} + \frac{1}{\left(r^2 + \left(\frac{d}{2} - \boldsymbol{z}\right)^2\right)^{3/2}}\right\}$$

b) When one sets $d = r$ one finds

$$\vec{B}_{total} = \frac{\mu_0 N I r^2 \hat{k}}{2}\left\{\frac{1}{\left(r^2 + \left(\frac{r}{2} + \boldsymbol{z}\right)^2\right)^{3/2}} + \frac{1}{\left(r^2 + \left(\frac{r}{2} - \boldsymbol{z}\right)^2\right)^{3/2}}\right\}$$

I suppose you could leave it as is, but I notice I can factor out an r^3 from the denominators…

$$\vec{B}_{total} = \frac{\mu_0 N I r^2 \hat{k}}{2}\left\{\frac{1}{\left(r^2 + r^2\left(\frac{1}{2} + \frac{\boldsymbol{z}}{\boldsymbol{r}}\right)^2\right)^{3/2}} + \frac{1}{\left(r^2 + r^2\left(\frac{1}{2} - \frac{\boldsymbol{z}}{\boldsymbol{r}}\right)^2\right)^{3/2}}\right\}$$

$$\vec{B}_{total} = \frac{\mu_0 N I r^2 \hat{k}}{2}\left\{\frac{1}{r^3\left(1 + \left(\frac{1}{2} + \frac{\boldsymbol{z}}{\boldsymbol{r}}\right)^2\right)^{3/2}} + \frac{1}{r^3\left(1 + \left(\frac{1}{2} - \frac{\boldsymbol{z}}{\boldsymbol{r}}\right)^2\right)^{3/2}}\right\}$$

$$\vec{B}_{total} = \frac{\mu_0 N I \hat{k}}{2r}\left\{\frac{1}{\left(1 + \left(\frac{1}{2} + \frac{\boldsymbol{z}}{\boldsymbol{r}}\right)^2\right)^{3/2}} + \frac{1}{\left(1 + \left(\frac{1}{2} - \frac{\boldsymbol{z}}{\boldsymbol{r}}\right)^2\right)^{3/2}}\right\}$$

$$\vec{B}_{total} = \frac{\mu_0 N I \hat{k}}{2r}\left\{\frac{1}{\left(\frac{5}{4} + \frac{z}{r} + \frac{z^2}{r^2}\right)^{3/2}} + \frac{1}{\left(\frac{5}{4} - \frac{z}{r} + \frac{z^2}{r^2}\right)^{3/2}}\right\}$$

Not much better, but perhaps slightly easier to work with.

Solution continues on next page…

c) At the origin we set $z = 0$ to find

$$\vec{B}_{total} = \frac{\mu_0 N I \hat{k}}{2r}\left\{\frac{1}{\left(\frac{5}{4}\right)^{3/2}} + \frac{1}{\left(\frac{5}{4}\right)^{3/2}}\right\}$$

$$\vec{B}_{total} = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N I \hat{k}}{r}$$

d) I will take the derivative using the formula

$$\vec{B}_{total} = \frac{\mu_0 N I r^2 \hat{k}}{2}\left\{\frac{1}{(r^2 + (d + \boldsymbol{z})^2)^{3/2}} + \frac{1}{(r^2 + (d - \boldsymbol{z})^2)^{3/2}}\right\}$$

$$\frac{d}{dz}\vec{B}_{total} = \frac{\mu_0 N I r^2 \hat{k}}{2}\frac{d}{dz}\left\{\frac{1}{(r^2 + (d + \boldsymbol{z})^2)^{3/2}} + \frac{1}{(r^2 + (d - \boldsymbol{z})^2)^{3/2}}\right\}$$

$$\frac{d}{dz}\vec{B}_{total} = \frac{\mu_0 N I r^2 \hat{k}}{2}\left\{\frac{d}{dz}(r^2 + (d + \boldsymbol{z})^2)^{-3/2} + \frac{d}{dz}(r^2 + (d - \boldsymbol{z})^2)^{-3/2}\right\}$$

$$\frac{d}{dz}\vec{B}_{total} = \frac{\mu_0 N I r^2 \hat{k}}{2}\left\{\left(-\frac{3}{2}\right)(r^2 + (d + \boldsymbol{z})^2)^{-\frac{5}{2}}\left(\frac{d}{dz}(d + \boldsymbol{z})^2\right) + \left(-\frac{3}{2}\right)(r^2 + (d - \boldsymbol{z})^2)^{-\frac{5}{2}}\left(\frac{d}{dz}(d - \boldsymbol{z})^2\right)\right\}$$

$$\frac{d}{dz}\vec{B}_{total} = \frac{\mu_0 N I r^2 \hat{k}}{2}\left\{\left(-\frac{3}{2}\right)(r^2 + (d + \boldsymbol{z})^2)^{-\frac{5}{2}}(2)(d + \boldsymbol{z})(+1) + \left(-\frac{3}{2}\right)(r^2 + (d - \boldsymbol{z})^2)^{-\frac{5}{2}}(2)(d - \boldsymbol{z})(-1)\right\}$$

$$\frac{d}{dz}\vec{B}_{total} = \frac{\mu_0 N I r^2 \hat{k}}{2}\left\{\frac{-3(d + \boldsymbol{z})}{(r^2 + (d + \boldsymbol{z})^2)^{-\frac{5}{2}}} + \frac{+3(d - \boldsymbol{z})}{(r^2 + (d - \boldsymbol{z})^2)^{-\frac{5}{2}}}\right\}$$

Now we can set $z = 0$ to find

$$\frac{d}{dz}\vec{B}_{total} = \frac{\mu_0 N I r^2 \hat{k}}{2}\left\{\frac{-3d}{(r^2 + d^2)^{-\frac{5}{2}}} + \frac{+3d}{(r^2 + d^2)^{-\frac{5}{2}}}\right\}$$

$$\frac{d}{dz}\vec{B}_{total} = 0$$

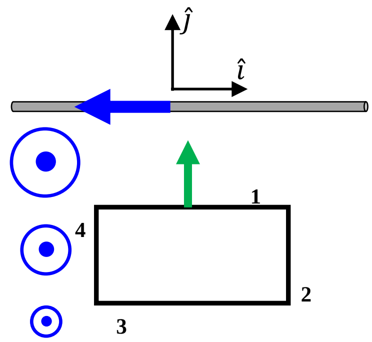
Going further: if you are particularly masochistic, you can discover $\frac{d^2\vec{B}_{total}}{dz^2} = 0$ & $\frac{d^3\vec{B}_{total}}{dz^3} = 0$ **WHEN $\boldsymbol{d = R}$**!!!

Why care? These results show $\vec{B}_{total}$ is *extremely* uniform inside Helmholtz coils (midway between the coils). Helmholtz coils can produce somewhat strong fields that are nearly uniform while also making it easy to access the region of magnetic field. It is also easy to vary the size of the field (by varying the current delivered to the coils).

Helmholtz coils are useful for doing experiments which require a uniform external magnetic field.

29.20

a) **Two alternative solutions are provided below this first solution:**



The long straight wire is the source of $\vec{B}_{ext}$ for the rectangular loop.
The *magnitude* of the field created by the straight wire is

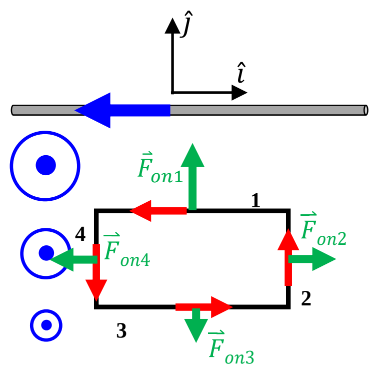
$$B_{ext} = \frac{\mu_0 I}{2\pi y}$$

where $r = y$ is distance from the straight wire (for this case).
Using the right hand rule for straight wires gives the direction.

$$\vec{B}_{ext} = \frac{\mu_0 I}{2\pi y}\hat{k}$$

One expects the force on segments 2 & 4 to be of equal size but opposite directions (regardless of current direction in the loop).
We expect the forces on segments 1 & 3 to also be in opposite directions.
HOWEVER we expect the force on segment 1 to be stronger than the force on segment 3.
This is because the field is stronger at segment 1's position.

Since we are told the *net* force on the loop is upwards, we know the force on *segment 1* must be upwards.
I can now use trial and error on segment 1 to figure out which direction (for current flow in the loop) causes an upwards force on segment 1.
I tried both clockwise and counter-clockwise loop currents.
Current in the loop must flow COUNTER-clockwise.



Alternative solution 1:
Magnetic fields produced by currents tend to align themselves with each other.
The magnetic field produced by the loop points either into or out of the page (at every point in the plane of the figure).
Since we are told the loop is attracted to the straight wire, we know it must be producing a field oriented the same direction as the long straight wire.
This occurs when current in the loop flows COUNTER-clockwise.

Alternative solution 2:
Segment 1 is the closet to the long straight wire, we know the force on it dominates the force on the loop.
Therefore, we need the force on segment 1 to point upwards.
We know like charges repel…but like currents attract.
This implies current in the loop flows COUNTER-clockwise.

b) There is zero torque on the loop.
The force on each segment points away from the center of the loop.
The loop is under tension.

Solution continues on the next page…

c) All of segment 1 is distance $y = d$ from the straight wire.
This implies

$$\vec{B}_{ext} = \left(\frac{\mu_0 I}{2\pi d}\hat{k}\right)$$

Notice $\vec{B}_{ext}$ is uniform along the entire length of segment 1.
We don't need to integrate to find the force.

$$\vec{F}_{on1} = I_{loop}\vec{L}_1 \times \vec{B}_{ext}$$

I'm assuming the longer dimension of the loop in the figure implies length $2d$.

$$\vec{F}_{on1} = I(-2d\hat{\imath}) \times \left(\frac{\mu_0 I}{2\pi d}\hat{k}\right)$$

$$\boldsymbol{\vec{F}_{on1} = \frac{\mu_0 I^2}{\pi}\hat{\jmath}}$$

Segment 3 is twice as far from the wire which cuts $\vec{B}_{ext}$ by a factor of 2 (direction of $\vec{B}_{ext}$ unchanged).
The current runs the other direction (which flips the force direction).

$$\boldsymbol{\vec{F}_{on2} = -\frac{\mu_0 I^2}{2\pi}\hat{\jmath}}$$

For segment 2 one uses

$$\vec{F}_{on2} = \int_{y=d}^{y=2d} d\vec{s} \times \vec{B}_{ext}$$

$$\vec{F}_{on2} = \int_{y=d}^{y=2d} (I\ dy\ \hat{\jmath}) \times \left(\frac{\mu_0 I}{2\pi y}\hat{k}\right)$$

$$\vec{F}_{on2} = \frac{\mu_0 I^2}{2\pi}\int_{y=d}^{y=2d} \frac{dy}{y}(\hat{\jmath} \times \hat{k})$$

$$\boldsymbol{\vec{F}_{on2} = \frac{\mu_0 I^2 \ln 2}{2\pi}\hat{\imath}}$$

$$\boldsymbol{\vec{F}_{on2} \approx 0.347\frac{\mu_0 I^2}{\pi}\hat{\imath}}$$

Finally, one expects the force on segment 4 should be equal in size, but opposite in direction to, the force on segment 2.

$$\boldsymbol{\vec{F}_{on4} \approx -0.347\frac{\mu_0 I^2}{\pi}\hat{\imath}}$$

Note: a student suggested this integral should actually be

$$\vec{F}_{on2} = \int_{y=-2d}^{y=-d} d\vec{s} \times \vec{B}_{ext}$$

When working the integral out this way one gets $\ln\frac{1}{2}$ instead of $\ln 2$.

While at first glance this seems to cause serious problems, keep in mind $\ln\frac{1}{2} = \ln 2^{-1} = -\ln 2$.

Doing the integral this way implies a sign flip which could always be checked using a right hand rule…

29.20½ Consider the augmented figure shown below.

a) I used the right hand rule to determine the direction of the positive z-axis.
I curl my fingers in the direction of current to determine the direction of the magnetic moment vector.
Notice that at all points in the plane of the loop, if you look very carefully at each iron filing, every magnetic field vector is pointing in the $-\hat{k}$ direction.

How can I tell the difference between $\pm\hat{k}$? I can't…but I know the right hand rule for the direction of $\vec{B}$ at the center of a coil and I use that to determine the direction.

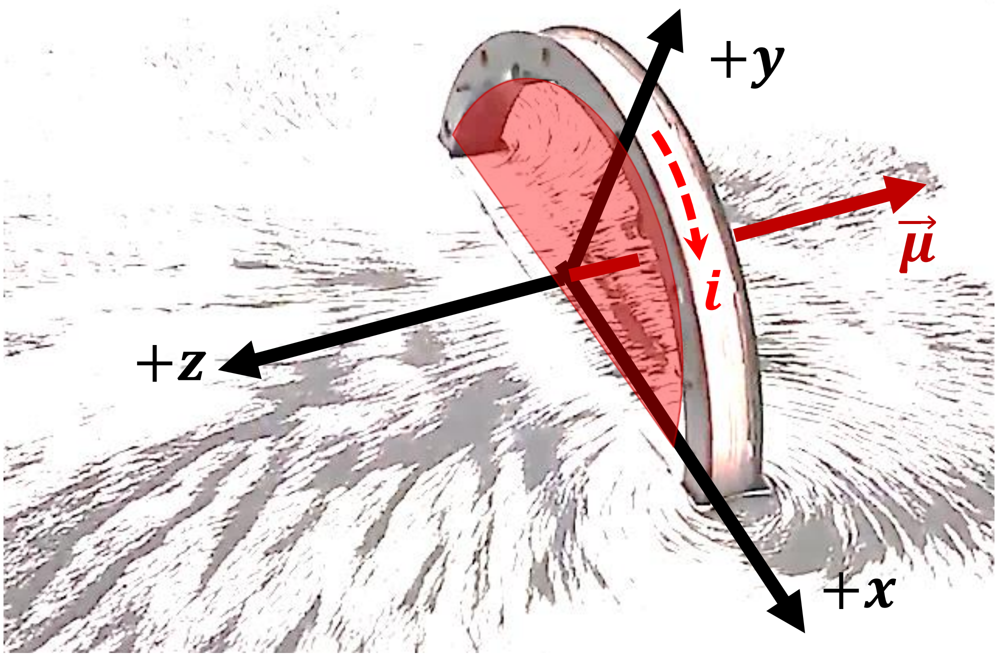
b) For a flat surface, we are free to choose the either direction perpendicular to the surface. To me it makes the most sense to say the area vector of the loop points the same direction as the magnetic moment vector (in this case $-\hat{k}$).

c) Magnetic flux is non-zero since

1) $\vec{B}$ & $\vec{A}$ point the same way and

2) $\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B\cos\theta_{BA}\ dA$

If $\vec{B}$ & $\vec{A}$ point the same way the angle $\theta_{BA} = 0°$ which implies $\cos\theta_{BA} = 1$. The integral should turn out non-zero.



29.21

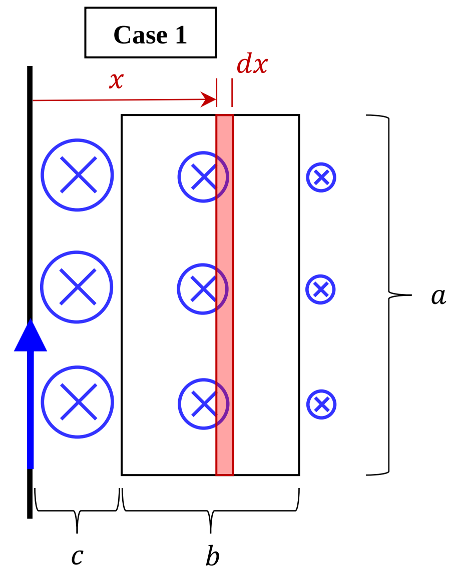
a) With respect to *angle*, max mag flux occurs when the loop area vector and $\vec{B}_{ext}$ point in the same direction. This occurs for cases 1 & 2.

Because $\vec{B}_{ext}$ gets weaker farther from the straight wire, we expect Case 1 should have more field lines penetrating it (compared to Case 2).

Case 1 experiences the largest magnetic flux.

b) The magnetic flux computation is

$$\Phi_B = \int_i^f \vec{B}_{ext} \cdot d\vec{A}$$



Since we have a *flat* loop, we are free to choose the *direction* of the area vector as we see fit. For *curved* loops, one generally chooses radially outwards as the surface direction. In this case, I choose to have the area vector direction match the direction of $\vec{B}_{ext}$.

$$\Phi_B = \int_c^{c+b} \left(-\frac{\mu_0 I}{2\pi x}\hat{k}\right) \cdot \left(a\,dx(-\hat{k})\right)$$

$$\Phi_B = \int_c^{c+b} \frac{\mu_0 I a}{2\pi x}\,dx$$

$$\Phi_B = \frac{\mu_0 I a}{2\pi} \ln\left|\frac{c+b}{c}\right|$$

$$\boldsymbol{\Phi_B = \frac{\mu_0 I a}{2\pi} \ln\left|1+\frac{b}{c}\right|}$$

Note: this type of calculation relates to electrical power generation.

c) If we redid the computation for Case 2 the area vector and the limits would change.

$$\Phi_B = \int_c^{c+a} \left(-\frac{\mu_0 I}{2\pi x}\hat{k}\right) \cdot \left(b\,dx(-\hat{k})\right)$$

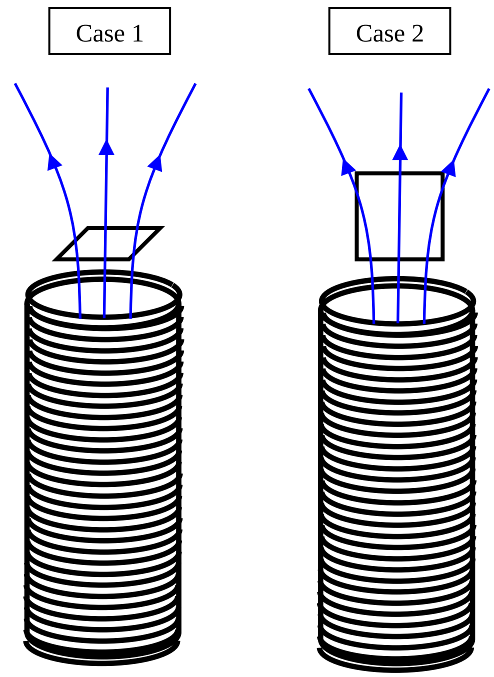
$$\boldsymbol{\Phi_B = \frac{\mu_0 I b}{2\pi} \ln\left|1+\frac{a}{c}\right|}$$

d) Yes, flux is zero in both of those cases!
In cases 3 & 4, I'm assuming the center of each loop is aligned with the straight wire.
As such, these loop experience zero NET flux.
Some field lines curve *into* the *front* half of each loop, but an equal number curve *out* of the *back* half.

Problem 29.22

To determine the direction of the solenoid's field, use the right hand rule for coils of wire.

1. Curl the fingers of your right hand in the direction of current in the coil.
2. Thumb points in the direction of the filed created by the coil.



We know the field of the solenoid is

$$B_{solenoid} = \mu_0 nI$$

in the center of the solenoid.

At the end of the solenoid, we are told

$$B_{ext} = \frac{1}{2} B_{solenoid} = \frac{\mu_0 nI}{2}$$

Going away from the end of the solenoid, one expect the field lines to warp. That said, at all points in the square loop, just above the end of the solenoid, I am going to assume $\vec{B}_{ext}$ is

1. directed upwards
2. *approximately* constant in size

This simplifies the flux calculation immensely.

$$\Phi_B = \int_i^f \vec{B}_{ext} \cdot d\vec{A}$$

I choose to orient the area vector of the flat loop in Case 1 parallel to $\vec{B}_{ext}$.

$$\Phi_B = \int_i^f B_{ext}\, dA$$

$$\Phi_B = \int_i^f \left(\frac{\mu_0 nI}{2}\right)$$

$$\Phi_B = \frac{\mu_0 nI}{2} \int_i^f dA$$

$$\Phi_B = \frac{\mu_0 nI}{2} A$$

We are told the main diagonal of the square loop matches the diameter of the solenoid.
The main diagonal of a square loop has length $s\sqrt{2}$ (where s is the side length of the square loop).
Therefore

$$s\sqrt{2} = 2R$$

$$s = \frac{2}{\sqrt{2}} R$$

$$A = s^2$$

$$A = 2R^2$$

$$\boldsymbol{\Phi_B = \mu_0 nIR^2}$$

For Case 2, flux is ZERO.

The are vector is perpendicular to $\vec{B}_{ext}$ at all points within the loop.

29.22 Bonus Problem Solution continues on the next page…

29.22 Bonus Problem

We are asked to think about the net force and torque on each loop.
Be careful, there can only be a (magnetic) force or torque on the loop if we assume current runs in the loop.
Also, net (magnetic) force on any loop is zero if $\vec{B}_{ext}$ is *uniform*.
In this case, the magnetic field is definitely non-uniform.

This thought experiment is probably easiest to analyze by thinking about the magnetic moment $\vec{\mu}$ of the loop.

If $\vec{\mu}$ in Case 1 is directed *upwards*, the field produced by the loop *matches* the field produced by the solenoid.
The loop is *attracted* to the solenoid by the magnetic force and there is zero magnetic torque.

If $\vec{\mu}$ in Case 1 is directed *downwards*, the field produced by the loop *opposes* the field produced by the solenoid.
The loop is *repelled* by the solenoid by the magnetic force and there is zero magnetic torque.

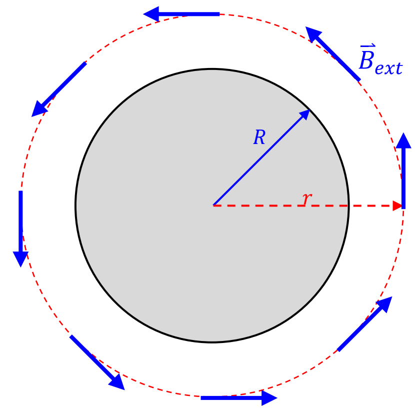
If $\vec{\mu}$ in Case 2 is directed *into the page*, the magnetic torque on the loop should try to align $\vec{\mu}$ with $\vec{B}_{ext}$.
This causes a torque in the $+\hat{\imath}$ direction (*axis* of torque is to the right).
Note: the coil feels a slightly larger force *into* the page on the bottom segment than it does *out of* the page on the top segment.
The net force is non-zero…but probably close to zero.
It pushes the coil's center of mass slightly *into* the page.

If $\vec{\mu}$ in Case 2 is directed *out of the page*, the magnetic torque on the loop should try to align $\vec{\mu}$ with $\vec{B}_{ext}$.
This causes a torque in the $-\hat{\imath}$ direction (*axis* of torque is to the left).
Note: the coil feels a slightly larger force *out of* the page on the bottom segment than it does *into* the page on the top segment.
The net force is non-zero…but probably close to zero.
It pushes the coil's center of mass slightly *out of* the page.

29.23

Case 1:

$B_{r>R}$
End View of Shell



a) & b) Are shown at right. We are told current runs out of the page (producing the B-field shown at right).

c) Notice $\vec{B}$ always points tangent to the circular path.
If we choose to travel around the circular loop in clockwise fashion, this implies $\vec{B}$ is parallel to $d\vec{s}$).
This implies

$$\oint_i^f \vec{B} \cdot d\vec{s} = \oint_i^f B\, ds$$

All points on the loop are equidistant from the wire.
Therefore the magnitude of $\vec{B}$ is constant as we travel around the loop.

$$\oint_i^f B\, ds = B \oint_i^f ds = Bs$$

d) All current is enclosed.

e) The math is as follows

$$Bs = \mu_0 I_{enc}$$

$$B2\pi r = \mu_0(all\ of\ it)$$

$$B2\pi r = \mu_0 I$$

$$\boldsymbol{B = \frac{\mu_0 I}{2\pi r}}$$

Note: we typically assume $\hat{\theta} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$ implies going counter-clockwise around a circle.
Since, in this case, the field is circling the wire counter-clockwise

$$\boldsymbol{\vec{B} = \frac{\mu_0 I}{2\pi r}(+\hat{\theta})}$$

Worth noting: remember r is free to vary (different Amperian loop sizes could have been chosen).
On the contrary, R is a constant (the size of the wire is *not* changing).

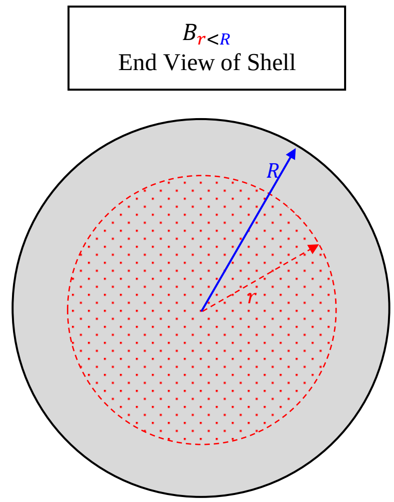
f) Units look good.

Case 2 on the next page…

29.23 continued

Case 2:

a) See figure at right.



b) For a *uniformly* distributed current

$$J = \frac{I_{total}}{A_{total}}$$

$$[J] = \frac{[I_{total}]}{[A_{total}]}$$

$$[J] = \frac{\text{A}}{\text{m}^2}$$

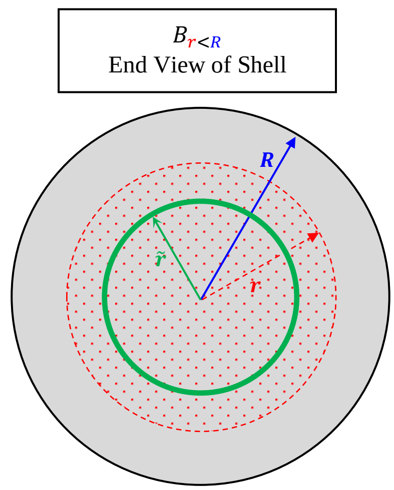
c) Because current is *uniformly* distributed

$$J = \frac{I_{total}}{A_{total}}$$

$$J = \frac{I}{\pi R^2}$$

d) Consider the lower figure at right.
The goal is to compute contributions to the total current by computing how much current passes through each ring of radius $\tilde{r}$.
Notice this ring has a <u>cross-sectional</u> area

$$dA = 2\pi\tilde{r}\, d\tilde{r}$$



This in turns gives

$$Bs = \mu_0 I_{enc}$$

$$Bs = \mu_0 \int_0^r J\, dA$$

$$Bs = \mu_0 \int_0^r \frac{I}{\pi R^2}\, 2\pi\tilde{r}\, d\tilde{r}$$

$$Bs = \frac{2\mu_0 I}{R^2} \int_0^r \tilde{r}\, d\tilde{r}$$

$$Bs = \frac{2\mu_0 I}{R^2} \left[\frac{\tilde{r}^2}{2}\right]_0^r$$

$$Bs = \frac{\mu_0 I r}{R^2}$$

Recall, here $s = 2\pi r$ is the total length around the Amperian loop.

$$B(2\pi r) = \frac{\mu_0 I r^2}{R^2}$$

$$\boldsymbol{B = \frac{\mu_0 I r}{2\pi R^2}} \quad \rightarrow \quad \boldsymbol{\vec{B} = \frac{\mu_0 I r}{2\pi R^2}}(+\hat{\boldsymbol{\theta}})$$

The units check.
Also, notice the two formulas (from Case 1 & Case 2) match at the boundary $r = R$.
Remember r is free to vary (different Amperian loop sizes could have been chosen).
On the contrary, R is a constant (the size of the wire is *not* changing).

Solution continues on the next page…

29.23 continued

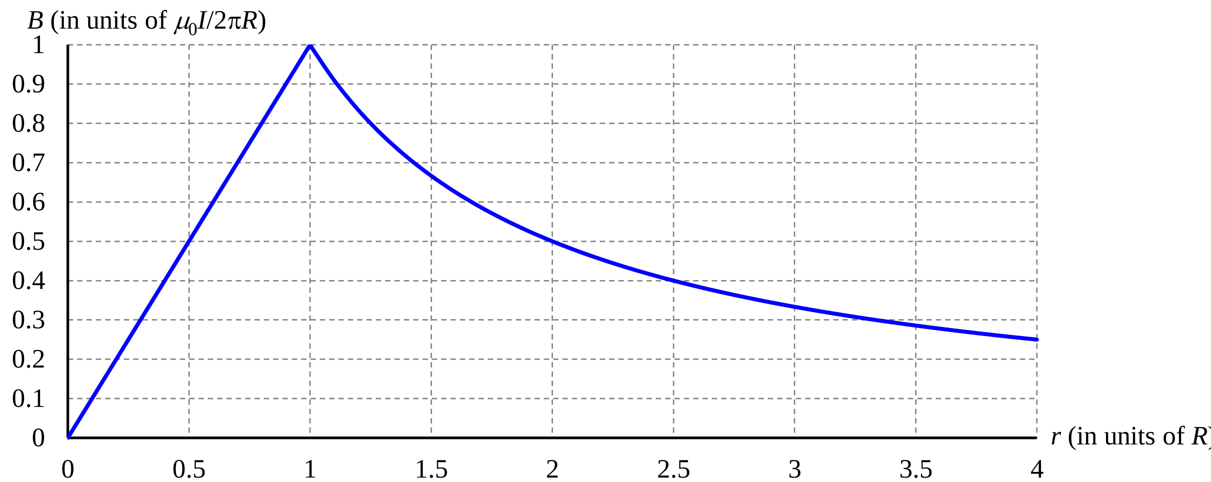
Finally, we make the graph and sketch the field around the wire.

Tip: it is much easier to make the plot by assuming the following two things:

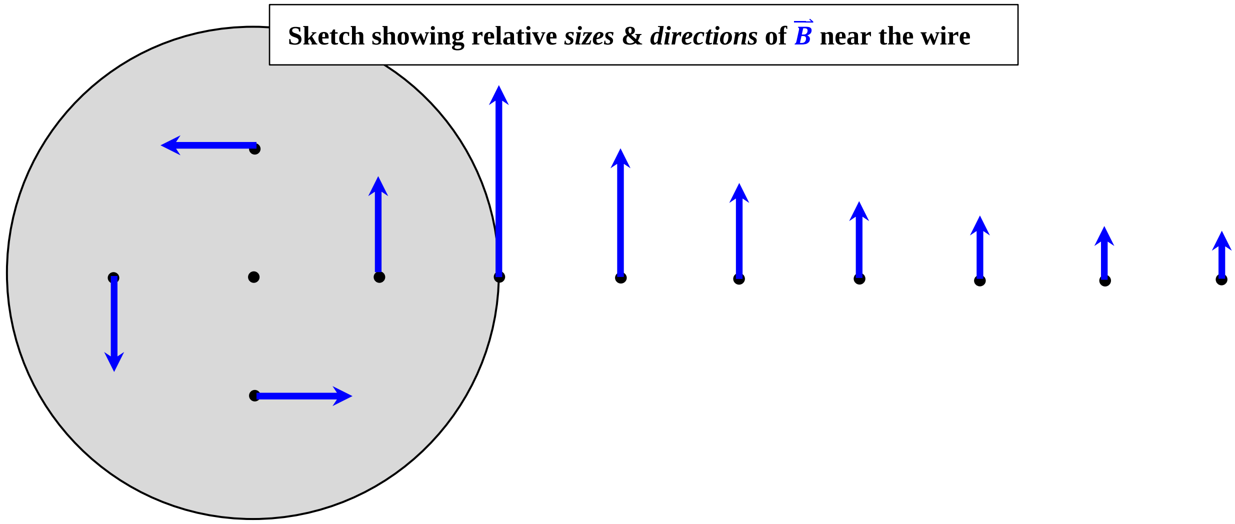
1. Use $\frac{\mu_0 I}{2\pi} = 1$.
2. Use $R = 1$.

If we make these assumptions, the field (magnitude) formulas become

For $r < R$	For $r > R$
$B = \frac{\mu_0 I r}{2\pi R^2}$ $B = r \quad \left(\text{in units of } \frac{\mu_0 I}{2\pi R}\right)$	$B = \frac{\mu_0 I}{2\pi r}$ $B = \frac{1}{r} \quad \left(\text{in units of } \frac{\mu_0 I}{2\pi R}\right)$

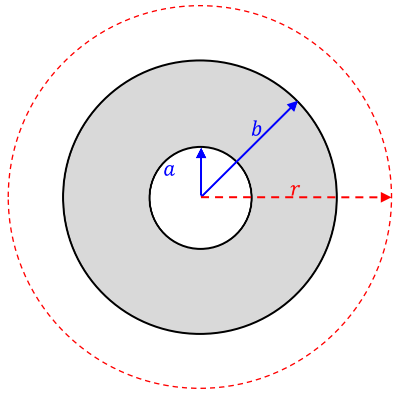
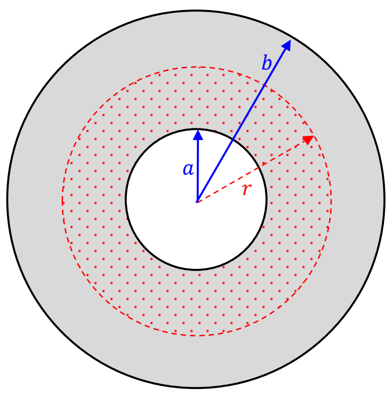


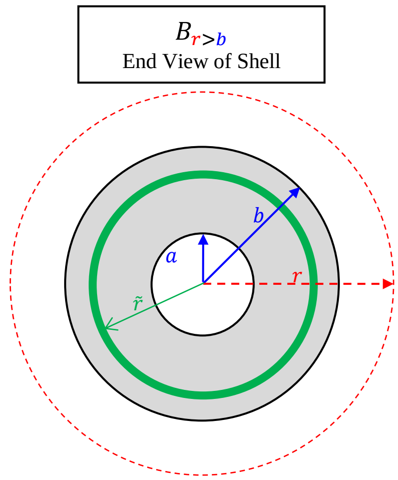
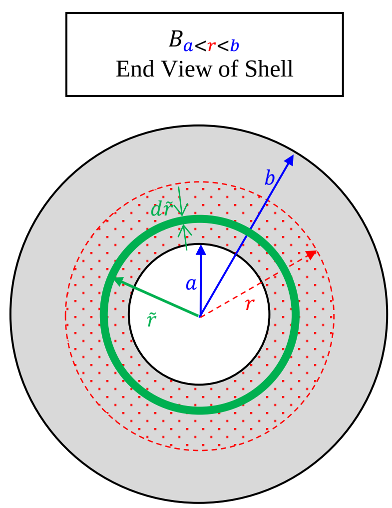
Sketch showing relative *sizes* & *directions* of $\vec{B}$ near the wire



Ampere's law in cylindrical symmetry shortcuts/summary

Uniform cases on this page, non-uniform on next page. Nested cylindrical shells (coax cable) to follow.

Conditions	Picture	Equations
Uniform, outside shell	$B_{r>b}$ End View of Shell 	$Bs = \mu_0 I_{enc}$ $B2\pi r = \mu_0(all\ of\ it)$ $B2\pi r = \mu_0 I$ $B = \frac{\mu_0 I}{2\pi r}$
Uniform, inside shell	$B_{a<r<b}$ End View of Shell 	$Bs = \mu_0 I_{enc}$ $B2\pi r = \mu_0(some\ of\ it)$ $B2\pi r = \mu_0 I\left(\frac{A_{Amperian}}{A_{Total}}\right)$ $B2\pi r = \mu_0 I\left(\frac{\pi r^2 - \pi a^2}{\pi b^2 - \pi a^2}\right)$ $B = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$

Conditions	Picture	Equations
Non-uniform, outside shell For this example let $J = \alpha r^7$ $I_{total} = \int_a^b J\,dA$ $I_{total} = \int_a^b \alpha\tilde{r}^7\,(2\pi\tilde{r}\,d\tilde{r})$ $I_{total} = 2\pi\alpha\int_a^b \tilde{r}^8\,d\tilde{r}$ $I_{total} = \frac{2\pi\alpha}{9}(b^9 - a^9)$ Therefore $\boldsymbol{\alpha = \frac{9I_{total}}{2\pi(b^9 - a^9)}}$		$Bs = \mu_0 I_{enc}$ $B2\pi r = \mu_0(all\ of\ it)$ $B2\pi r = \mu_0 I$ $B2\pi r = \mu_0\int_a^b J\,dA$ $B2\pi r = \mu_0\int_a^b \alpha\tilde{r}^7\,(2\pi\tilde{r}\,d\tilde{r})$ $B2\pi r = 2\pi\alpha\mu_0 \left.\frac{r^9}{9}\right\|_a^b$ $B = \frac{\alpha\mu_0}{9r}(b^9 - a^9)$ Note: if we eliminate α using result shown in left column $B = \left(\frac{9I_{total}}{2\pi(b^9 - a^9)}\right)\frac{\mu_0}{9r}(b^9 - a^9)$ $\boldsymbol{B = \frac{\mu_0 I_{total}}{2\pi r}}$
Non-uniform, inside shell For this example let $J = \alpha r^7$ $I_{total} = \int_a^b J\,dA$ $I_{total} = \int_a^b \alpha\tilde{r}^7\,(2\pi\tilde{r}\,d\tilde{r})$ $I_{total} = 2\pi\alpha\int_a^b \tilde{r}^8\,d\tilde{r}$ $I_{total} = \frac{2\pi\alpha}{9}(b^9 - a^9)$ Therefore $\boldsymbol{\alpha = \frac{9I_{total}}{2\pi(b^9 - a^9)}}$		$Bs = \mu_0 I_{enc}$ $B2\pi r = \mu_0(some\ of\ it)$ $B2\pi r = \mu_0 I$ $B2\pi r = \mu_0\int_a^r J\,dA$ $B2\pi r = \mu_0\int_a^r \alpha\tilde{r}^7\,(2\pi\tilde{r}\,d\tilde{r})$ $B2\pi r = 2\pi\alpha\mu_0 \left.\frac{r^9}{9}\right\|_a^r$ $B = \frac{\alpha\mu_0}{9r}(r^9 - a^9)$ Note: if we eliminate α using result shown in left column $B = \left(\frac{9I_{total}}{2\pi(b^9 - a^9)}\right)\frac{\mu_0}{9r}(r^9 - a^9)$ $\boldsymbol{B = \frac{\mu_0 I_{total}}{2\pi r}\frac{(r^9 - a^9)}{(b^9 - a^9)}}$

29.24

a) Current density (magnitude) for a *uniform* shell is given by

$$J = \frac{I_{total}}{A_{total}}$$

$$J = \frac{I}{\pi(3R)^2 - \pi(R)^2}$$

$$J = \frac{I}{8\pi R^2}$$

b) For $r < R$ no current is enclosed… $\boldsymbol{B_{r<R} = 0}$.

For $r > 3R$, outside the entire shell, *all* current is enclosed.

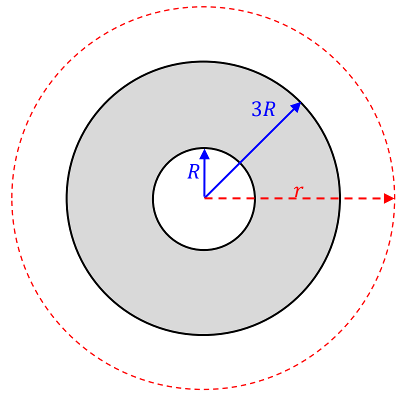
$$Bs = \mu_0 I_{enc}$$

$$B2\pi r = \mu_0(all\ of\ it)$$

$$B2\pi r = \mu_0 I$$

$$\boldsymbol{B_{r>3R} = \frac{\mu_0 I}{2\pi r}}$$

$B_{r>3R}$
End View of Shell



For $R < r < 3R$, the Amperian loop lies inside the shell.
When current is distributed uniformly enclosed current can be found using a ratio of the Amperian area to the total area of the shell.

$$Bs = \mu_0 I_{enc}$$

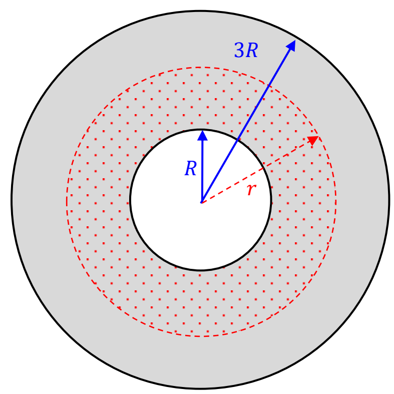
$$B2\pi r = \mu_0(some\ of\ it)$$

$$B2\pi r = \mu_0 I\left(\frac{A_{Amperian}}{A_{Total}}\right)$$

$$B2\pi r = \mu_0 I\left(\frac{\pi r^2 - \pi R^2}{\pi(3R)^2 - \pi R^2}\right)$$

$$\boldsymbol{B_{R<r<3R} = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - R^2}{8R^2}\right)}$$

$B_{R<r<3R}$
End View of Shell



Solution continues on the next page…

29.24c) A trick for making the plots is to set $\frac{\mu_0 I}{2\pi} = 1$ and $R = 1$.

Notice our formulas become

$$\boldsymbol{B_{r<R} = 0}$$

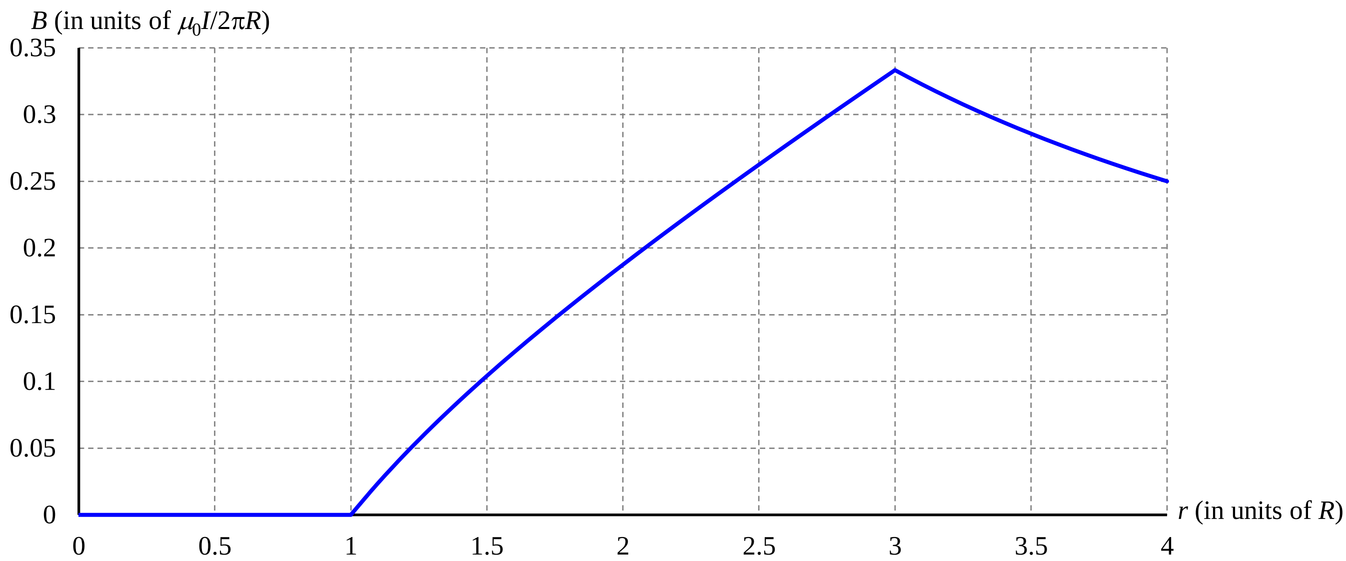
$$\boldsymbol{B_{R<r<3R} = \frac{1}{r}\left(\frac{r^2 - 1}{8}\right) = \frac{1}{8}\left(r - \frac{1}{r}\right)}$$

$$\boldsymbol{B_{r>3R} = \frac{1}{r}}$$

Checking the concavity of the second term can now be easily done.

Remember, we only require the SIGN of the second derivative to determine concavity.

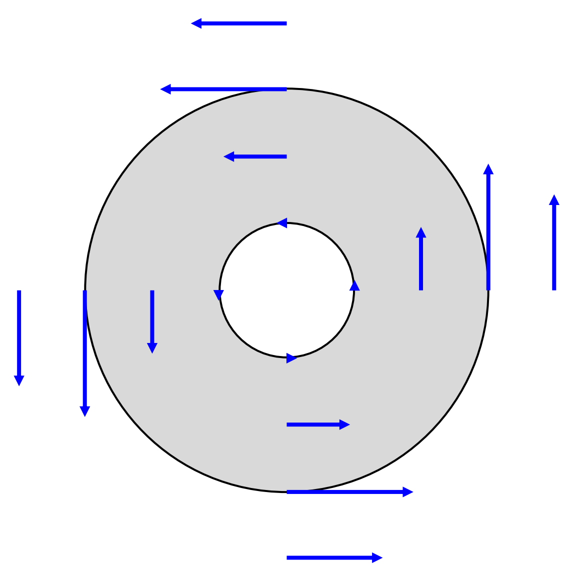
This term is concave *down* (negative 2nd derivative).



29.24d) The problem statement says current flows out of the page.
The right hand rule for straight wires helps:

1. Align right thumb with current direction (of straight wire).
2. Fingers of right hand curl around the wire in the direction of $\vec{B}$.

Based on this info, we expect the arrows to curl around the wire going counter-clockwise.
Using the plot above, I can estimate the relative sizes of the arrows.
My figure is shown at right.



29.25

a) Use the given current density formula to find the units

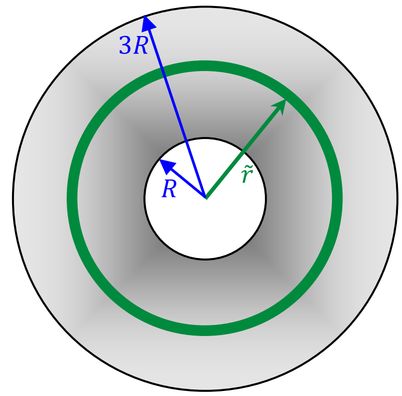
$$[J] = \frac{[\alpha]}{[r]}$$

$$[\alpha] = [J] \cdot [r]$$

$$[\alpha] = \frac{\mathrm{A}}{\mathrm{m}^2} \cdot \mathrm{m}$$

$$[\alpha] = \frac{\mathrm{A}}{\mathrm{m}}$$

b) We know <u>total</u> current I relates to density using



$$I = \int_i^f J\, dA$$

$$I = \int_R^{3R} \frac{\alpha}{\tilde{r}}\,(2\pi\tilde{r}\; d\tilde{r})$$

$$I = 2\pi\alpha \int_R^{3R} d\tilde{r}$$

$$I = 4\pi\alpha R$$

Rearranging to solve for α gives

$$\boldsymbol{\alpha = \frac{I}{4\pi R}}$$

c) We were told total current is I.

If the Amperian loop is drawn *outside* of the entire wire ($r > 3R$):

$$I_{enclosed} = I$$

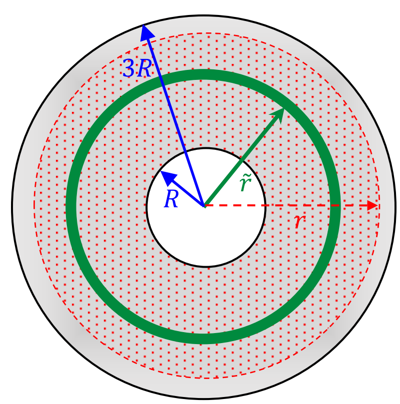
$$\boldsymbol{B_{r>3R} = \frac{\mu_0 I}{2\pi r}}$$

If the Amperian loop is *inside* the central air core ($r < R$):

$$I_{enclosed} = 0$$

$$\boldsymbol{B_{r<R} = 0}$$

If the Amperian loop is *inside* the shell ($R < r < 3R$):



$$I_{enclosed} = I$$

$$I_{enclosed} = \int_R^{r} \frac{\alpha}{\tilde{r}}\,(2\pi\tilde{r}\; d\tilde{r})$$

$$I_{enclosed} = 2\pi\alpha \int_R^{r} d\tilde{r}$$

$$I_{enclosed} = 2\pi\alpha(r - R)$$

To make comparing the formulas (and plotting) easier, we can use the result of part b $\left(\alpha = \frac{I}{4\pi R}\right)$.

$$I_{enclosed} = 2\pi\left(\frac{I}{4\pi R}\right)(r - R)$$

$$I_{enclosed} = \frac{I}{2R}(r - R)$$

$$\boldsymbol{B_{R<r<3R} = \frac{\mu_0 I}{2\pi r} \cdot \frac{(r - R)}{2R}}$$

Solution continues on the next page…

29.25 continued

In the air core	Inside the bulk of the shell	Outside the shell
$B_{r<R} = 0$	$B_{R<r<3R} = \frac{\mu_0 I}{2\pi r} \cdot \frac{(r-R)}{2R}$	$B_{r>3R} = \frac{\mu_0 I}{2\pi r}$

Verify the following:

- Does $B_{r<R} = B_{R<r<3R}$ when you plug in $r = R$ to both formulas?
- Does $B_{r>3R} = B_{R<r<3R}$ when you plug in $r = 3R$ to both formulas?

I verified it.

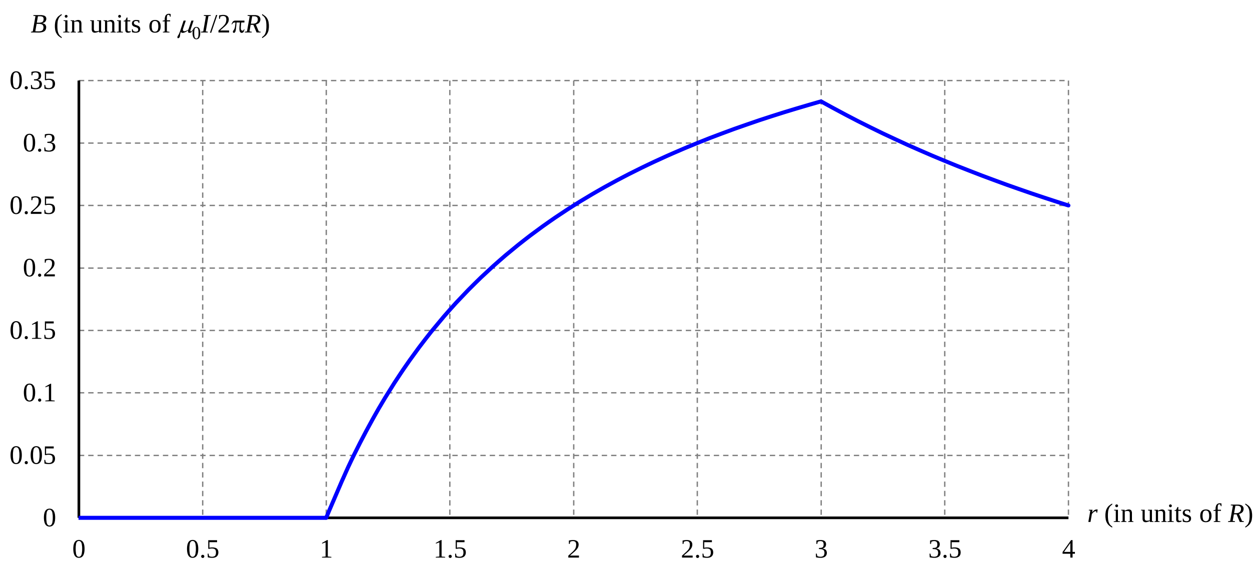
Now re-write these formulas using the standard trick of setting $\frac{\mu_0 I}{2\pi} = 1$ & $R = 1$:

In the air core	Inside the bulk of the shell	Outside the shell
$B_{r<R} = 0$	$B_{R<r<3R} = \frac{(r-1)}{2r} = \frac{1}{2} \cdot \left(1 - \frac{1}{r}\right)$	$B_{r>3R} = \frac{1}{r}$

Make a table of values (see figure at right).

r (in units of R)	B $\left(\text{in units of } \frac{\mu_0 I}{2\pi R}\right)$
0	0
1	0
2	$\frac{1}{4} = 0.250$
3	$\frac{1}{3} = 0.333$
4	$\frac{1}{4} = 0.250$

Check the concavity of any tricky terms.
The middle field is tricky.
Two derivatives of the constant terms will drop out.
The sign is thus determined by two derivatives of the $-\frac{1}{r}$ term.
The second derivative is *negative*…this implies concave *down*.



29.26

a) We know the units of a function are "no units".

Checking units of the current density formula gives

$$[J] = [\alpha][r][e^{-\beta r}]$$
$$\frac{\mathrm{A}}{\mathrm{m}^2} = [\alpha] \cdot \mathrm{m} \cdot (\text{no units})$$
$$[\boldsymbol{\alpha}] = \frac{\mathbf{A}}{\mathbf{m^3}}$$

We also know the units of the *argument* of a function are "no units".

This implies

$$[-\beta r] = \text{no units}$$

The minus sign has no effect on the units; said another way: $[-1] = \text{no units}$.

$$[\beta][r] = \text{no units}$$
$$[\beta] = \frac{\text{no units}}{[r]}$$
$$[\boldsymbol{\beta}] = \frac{\mathbf{1}}{\mathbf{m}}$$

b) For a *uniform* distribution of current one knows $J = \frac{I_{total}}{A_{total}}$…but *this* wire has *NON-UNIFORM* current density.

Use

$$I_{total} = \int_{inner\ radius}^{outer\ radius} J\ dA$$
$$I_{total} = \int_0^R (\alpha r e^{-\beta r})\ 2\pi r\ dr$$
$$I_{total} = 2\pi\alpha \int_0^R r^2 e^{-\beta r}\ dr$$

From the integral table, this one looks good if we set $a = -\beta$:

$$\int x^2\ e^{ax}\ dx = e^{ax}\left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)$$
$$I_{total} = 2\pi\alpha\left[e^{-\beta x}\left(-\frac{x^2}{\beta} - \frac{2x}{\beta^2} - \frac{2}{\beta^3}\right)\right]_0^R$$

At first glance it appears total current might be negative…yikes!

Have patience and be sure to use both limits…

$$I_{total} = 2\pi\alpha\left\{e^{-\beta R}\left(-\frac{R^2}{\beta} - \frac{2R}{\beta^2} - \frac{2}{\beta^3}\right) - e^{-0}\left(-\frac{0^2}{\beta} - \frac{0}{\beta^2} - \frac{2}{\beta^3}\right)\right\}$$
$$I_{total} = 2\pi\alpha\left\{\frac{2}{\beta^3} - e^{-\beta R}\left(\frac{R^2}{\beta} + \frac{2R}{\beta^2} + \frac{2}{\beta^3}\right)\right\}$$

We see the size of total current can be positive (as it should be) if

$$\frac{2}{\beta^3} > e^{-\beta R}\left(\frac{R^2}{\beta} + \frac{2R}{\beta^2} + \frac{2}{\beta^3}\right)$$

c) If the field at **P** points to the left, current in the wire must run INTO the page. Verify with the right hand rule.

d) *Outside* the wire ($r > R$) we know

$$B(2\pi r) = \mu_o I_{enclosed}$$

$$B(2\pi r) = \mu_o I_{total}$$

$$B_{r>R} = \frac{\mu_0}{2\pi r} \cdot 2\pi\alpha \left\{ \frac{2}{\beta^3} - e^{-\beta R} \left(\frac{R^2}{\beta} + \frac{2R}{\beta^2} + \frac{2}{\beta^3} \right) \right\}$$

$$B_{r>R} = \frac{\mu_0 \alpha}{r} \left\{ \frac{2}{\beta^3} - e^{-\beta R} \left(\frac{R^2}{\beta} + \frac{2R}{\beta^2} + \frac{2}{\beta^3} \right) \right\}$$

Inside the wire ($r < R$) we know

$$B(2\pi r) = \mu_o I_{enclosed}$$

$$B(2\pi r) = \mu_o 2\pi\alpha \int_0^r r^2 e^{-\beta r}\, dr$$

Notice the upper limit has changed.

From here I think you can do the rest of this part on your own.

e) I'll get to this plot when I have time. This one is nasty enough it is worth using a computer.

f) The question asks for the radius at which current density begins to decrease.
Take the derivative and set it equal to zero!

$$\frac{d}{dr} \alpha r e^{-\beta r} = 0$$

Constants out front won't affect the result.

$$\frac{d}{dr} r e^{-\beta r} = 0$$

$$e^{-\beta r} - \beta r e^{-\beta r} = 0$$

$$e^{-\beta r}(1 - \beta r) = 0$$

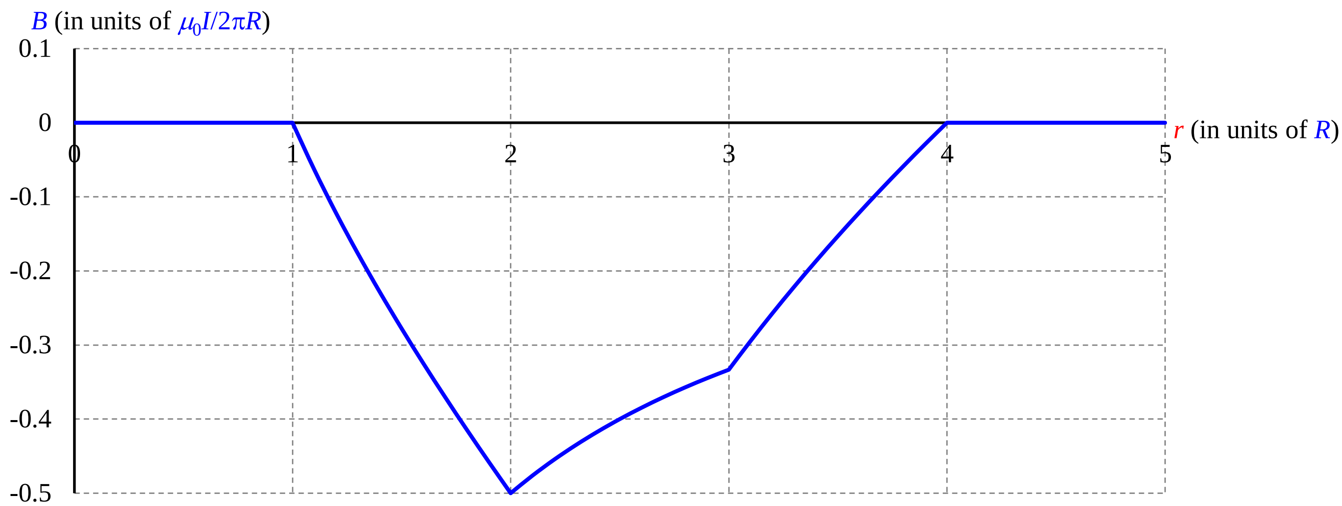
One finds $r = \frac{1}{\beta}$ is the special radius.

It would be interesting to test if total current can be positive when $r_{special} = \frac{1}{\beta} < R$ by plugging in $R = \frac{1}{\beta}$ into I_{total}.

29.27

$\boldsymbol{r < a}$	$\vec{\boldsymbol{B}}_{\boldsymbol{r<a}} = \boldsymbol{0}$
$\boldsymbol{a < r < 2a}$	$B_{a<r<2a} 2\pi r = \mu_0 I \left(\frac{A_{Amp}}{A_{Tot}}\right)$ $B_{a<r<2a} = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - a^2}{3a^2}\right)$ Current is running *into* the page (for the *inner shell*) $\vec{\boldsymbol{B}}_{\boldsymbol{a<r<2a}} = \frac{\boldsymbol{\mu_0 I}}{\boldsymbol{2\pi r}}\left(\frac{\boldsymbol{r^2 - a^2}}{\boldsymbol{3a^2}}\right)(-\hat{\boldsymbol{\theta}})$
$\boldsymbol{2a < r < 3a}$	$B 2\pi r = \mu_0 I$ $\vec{\boldsymbol{B}}_{\boldsymbol{2a<r<3a}} = \frac{\boldsymbol{\mu_0 I}}{\boldsymbol{2\pi r}}(-\hat{\boldsymbol{\theta}})$
$\boldsymbol{3a < r < 4a}$	$\vec{B}_{3a<r<4a} = \vec{B}_{inner\ shell} + \vec{B}_{outer\ shell}$ Since we are *outside* the *inner shell*, $\vec{B}_{inner\ shell} = \frac{\mu_0 I}{2\pi r}(-\hat{\theta})$ Since we are *inside* the *outer shell* $B_{outer\ shell} 2\pi r = \mu_0 I \left(\frac{A_{Amp}}{A_{Tot}}\right)$ $B_{outer\ shell} = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - 9a^2}{7a^2}\right)$ Current is running *out of* the page (for the *outer shell*) $B_{outer\ shell} = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - 9a^2}{7a^2}\right)(+\hat{\theta})$ Adding together gives $\vec{B}_{3a<r<4a} = \vec{B}_{inner\ shell} + \vec{B}_{outer\ shell}$ $\vec{B}_{3a<r<4a} = \frac{\mu_0 I}{2\pi r}(-\hat{\theta}) + \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - 9a^2}{7a^2}\right)(+\hat{\theta})$ $\vec{\boldsymbol{B}}_{\boldsymbol{3a<r<4a}} = \frac{\boldsymbol{\mu_0 I}}{\boldsymbol{2\pi r}}\left(\frac{\boldsymbol{r^2 - 9a^2}}{\boldsymbol{7a^2}} - \boldsymbol{1}\right)(+\hat{\boldsymbol{\theta}})$
$\boldsymbol{r > 4a}$	Outside of *both* shells $I_{enclosed} = I_1 + I_2 = 0$ Remember: one adds currents like vectors…even though current is *not* a vector. Physics is fun! $\vec{\boldsymbol{B}}_{\boldsymbol{r>4a}} = \boldsymbol{0}$

Plot of field VECTOR is on the next page…



To plot field MAGNTIUDE, take the absolute value of every point on the curve.

One sees the field magnitude plot would look like this one flipped upside down.

Going further: what would be different if the outer wire had twice the current of the inner wire?

Solutions on the next page…

Try to do it without looking…

Going further: what would be different if the outer wire had twice the current of the inner wire?

If we assumed the outer shell had twice the current…	
$r < a$	Outer shell has no effect on this region, result unchanged. $\vec{B}_{r<a} = 0$
$a < r < 2a$	Outer shell has no effect on this region, result unchanged. $\vec{B}_{a<r<2a} = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - a^2}{3a^2}\right)(-\hat{\theta})$
$2a < r < 3a$	Outer shell has no effect on this region, result unchanged. $\vec{B}_{2a<r<3a} = \frac{\mu_0 I}{2\pi r}(-\hat{\theta})$
$3a < r < 4a$	$\vec{B}_{3a<r<4a} = \vec{B}_{inner\ shell} + \vec{B}_{outer\ shell}$ $\vec{B}_{3a<r<4a} = \frac{\mu_0 I}{2\pi r}(-\hat{\theta}) + 2\frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - 9a^2}{7a^2}\right)(+\hat{\theta})$ $\vec{B}_{3a<r<4a} = \frac{\mu_0 I}{2\pi r}\left(2\frac{r^2 - 9a^2}{7a^2} - 1\right)(+\hat{\theta})$
$r > 4a$	Outside of *both* shells $I_{enclosed} = I_1 + I_2$ $I_{enclosed} = I\ (into\ page) + 2I(out\ of\ page)$ $I_{enclosed} = I(out\ of\ page)$ $\vec{B}_{r>4a} = \frac{\mu_0 I}{2\pi r}(+\hat{\theta})$