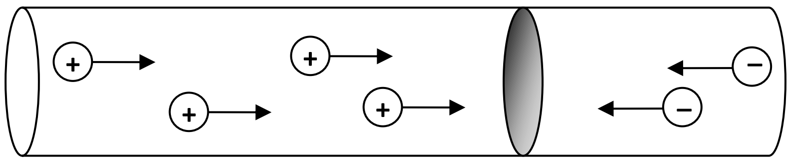
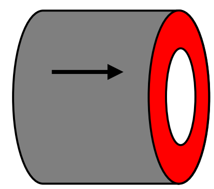
Chapter 26 Solutions

26.1 When thinking about current from a stream (or beam) of charges, pick a surface & a direction of travel through that surface. I will assume *to the right* as my direction through my surface. *Negatives moving left* are equivalent to *positives moving right* even though current is <u>not</u>, strictly speaking, a vector. The current is $6\,\frac{\text{nC}}{\text{s}} = 6$ nA going to the right.



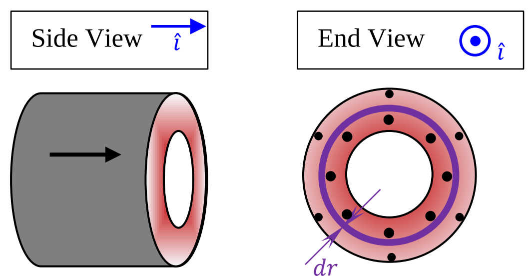
26.2 Answer: $\vec{J} = \frac{I}{3\pi R^2}\hat{\imath}$. Assumes to the right is standard $\hat{\imath}$ direction.



The red area is the cross-sectional area in this instance.

$$A = \pi R_{outer}^2 - \pi R_{inner}^2$$
$$A = \pi(2R)^2 - \pi R^2$$
$$A = 3\pi R^2$$

26.3 The pipe is shown at right.



While the shading is not perfect, I am trying to show the current density *decreasing* as radius *increases* according to

$$\vec{J} = \frac{\alpha}{r^2}\hat{\imath}$$

The black dots in the end view indicate current flowing out of the page (fewer black dots at larger radius).

Notice the *side view* indicates *to the right* is $\hat{\imath}$.

As a result, the *end view* indicate *out of the page* is $\hat{\imath}$.

$$I = \int_R^{2R} \vec{J}\cdot d\vec{A}$$
$$I = \int_R^{2R} \left(\frac{\alpha}{r^2}\hat{\imath}\right)\cdot(2\pi r\, dr\, \hat{\imath})$$
$$I = \int_R^{2R} \frac{2\pi\alpha}{r} dr$$
$$I = 2\pi\alpha \ln r \Big|_R^{2R}$$
$$I = 2\pi\alpha \ln 2$$

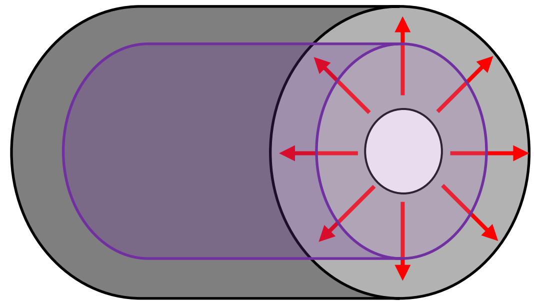
Think: check the units of every answer just in case.

From our original current density equation:

$$[\alpha] = [J]\cdot[r]^2 = \frac{\text{A}}{\text{m}^2}\cdot\text{m}^2 = \text{A}.$$

Units check!

26.4 When current flows radially towards the outer wall, the cross-sectional area current flows through is the *sidewall* (not the end cap) of a cylinder!



The sidewall area of an *arbitrary* cylinder (purple cylindrical shell in figure) of radius r and height h is $A_{sidewall} = 2\pi r h$.

For points *farther* from the center of the pipe (bigger values of r) we see the area of the sidewall *increases*.

Current *density* and area are inversely related.

We expect current *density* is greatest near the center of the pipe.

26.5

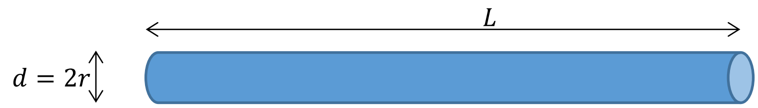
a) We are given $L = 1.00\text{ m}, r = 0.500\text{ mm}, \&\ \rho_{Au} = 24.4\text{ n}\Omega\cdot\text{m}$ (figure not to scale).

$$R = \frac{\rho L}{A}$$

$$R = \frac{\rho L}{\pi r^2}$$

$$R = 3.107 \times 10^{-2}\ \Omega$$

$$\boldsymbol{R = 31.07\text{ m}\Omega}$$



b) To *decrease* R by 5% one would want to make L 5% shorter.

c) Changing the radius is sneakier (because R depends on radius squared!).
If you are sure how to handle it, you can always take a ratio.
We want $R' = 0.95R$.

$$R' = 0.95R$$

$$\frac{\rho L}{\pi r'^2} = 0.95\frac{\rho L}{\pi r^2}$$

$$\frac{1}{r'^2} = 0.95\frac{1}{r^2}$$

$$r'^2 = \frac{r^2}{0.95}$$

$$r' = r\sqrt{\frac{1}{0.95}}$$

$$\boldsymbol{r' = 1.0260r}$$

Increase the radius by about 2.60% to decrease resistance by 5%!

d) The problem states we have gold (a metal) operating at 20.0℃.
For *metals*, increasing temperature typically increases resistance (opposite is typically true for *semi-conductors*).
There are two standard resistance versus temperature formulas:

$$R(T) = R_0(1 + \alpha\Delta T) \quad \textbf{or} \quad \Delta R = R_0\alpha\Delta T$$

It is important to know $R_0 = R_{@20}$℃ is resistance at the standard reference temperature of 20.0℃.

$$\Delta R = R_0\alpha\Delta T$$

$$-5\%\text{ of }R_0 = R_0\alpha\Delta T$$

$$-0.05R_0 = R_0\alpha\Delta T$$

$$-0.05 = \alpha\Delta T$$

$$\Delta T = -\frac{0.05}{\alpha}$$

$$T - 20.0℃ = -\frac{0.05}{\alpha}$$

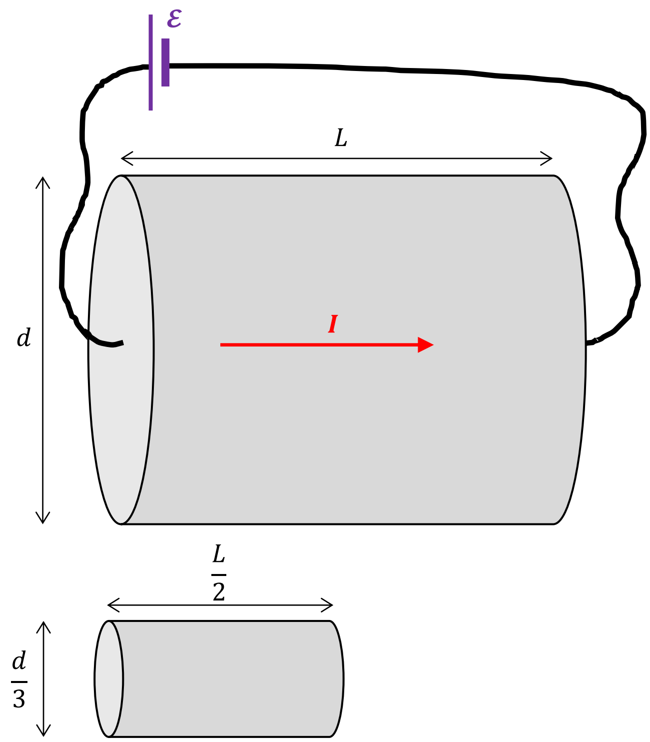
From a web search, I found the temperature coefficient of resistivity for gold as $\alpha_{Au} = 0.0034\frac{1}{℃}$.

$$\boldsymbol{T = 20.0℃ - \frac{0.05}{0.0034\frac{1}{℃}} = 5.29℃}$$

26.6

a) The figures show the dimensions as they relate to the words in the problem statement.

While not specifically stated, one typically assume current flows along the long axis of the wire (unless otherwise noted).



From Ohm's law we know

$$\Delta V = IR$$

$$R = \frac{\Delta V}{I}$$

$$\boldsymbol{R = \frac{\mathcal{E}}{I}}$$

Our resistivity equation tells us

$$R = \frac{\rho L}{A}$$

$$R = \frac{\rho L}{\pi r^2}$$

$$R = \frac{\rho L}{\pi \left(\frac{d}{2}\right)^2}$$

$$\boldsymbol{R = \frac{4\rho L}{\pi d^2}}$$

Combining these equations gives

$$\frac{\mathcal{E}}{I} = \frac{4\rho L}{\pi d^2}$$

$$\boldsymbol{\rho = \frac{\pi d^2 \mathcal{E}}{4LI}}$$

b) You could *probably* think deeply and come up with the answer… why not use a ratio or eqt'n to reduce effort?

$$I_{big} = I_{small}$$

$$\frac{\Delta V_{big}}{R_{big}} = \frac{\Delta V_{small}}{R_{small}}$$

$$\Delta V_{small} = \Delta V_{big} \frac{R_{small}}{R_{big}}$$

$$\Delta V_{small} = \mathcal{E} \frac{\frac{4\rho_{small} L_{small}}{\pi d_{small}^2}}{\frac{4\rho_{big} L_{big}}{\pi d_{big}^2}}$$

$$\Delta V_{small} = \mathcal{E} \frac{\frac{4\rho (L/2)}{\pi (d/3)^2}}{\frac{4\rho L}{\pi d^2}}$$

$$\boldsymbol{\Delta V_{small} = \frac{9}{2}\mathcal{E}}$$

Solution continues on the next page…

c) In this scenario, we are asked to determine the power ratio shown below:

$$power\ ratio = \frac{\mathcal{P}_{small}}{\mathcal{P}_{big}}$$

$$power\ ratio = \frac{i_{small}\Delta V_{small}}{i_{big}\Delta V_{big}}$$

Think: we were told in the previous part the two wires have identical current…the currents cancel!

$$power\ ratio = \frac{\Delta V_{small}}{\Delta V_{big}}$$

From the previous part we know $\Delta V_{big} = \mathcal{E}$ & $\Delta V_{small} = \frac{9}{2}\mathcal{E}$.

$$\boldsymbol{power\ ratio} = \mathbf{\frac{9}{2}}$$

<u>Another method</u> would be to write the power ratio as

$$power\ ratio = \frac{\mathcal{P}_{small}}{\mathcal{P}_{big}}$$

$$power\ ratio = \frac{i_{small}^2 R_{small}}{i_{big}^2 R_{big}}$$

Think: we were told in the previous part the two wires have identical current…the currents cancel!

$$power\ ratio = \frac{R_{small}}{R_{big}}$$

From the previous part we know $\frac{R_{small}}{R_{big}} = \frac{9}{2}$. We still find

$$\boldsymbol{power\ ratio} = \mathbf{\frac{9}{2}}$$

26.7

a) A $\text{mA} \cdot \text{hr}$ is a unit of *charge*.

$$2750 \times 10^{-3}\ \text{A} \cdot \text{hr} = 2750 \times 10^{-3} \frac{\text{C} \cdot \text{hr}}{\text{s}} = 2750 \times 10^{-3} \frac{\text{C} \cdot (3600\ \text{s})}{\text{s}} = 9900\ \text{C}$$

Strictly speaking, we say this battery is rated to move 9900 C of charge at potential 1.5 V.
WATCH OUT! It is probably better to think of batteries in $\text{mA} \cdot \text{hrs}$ than as storing charge.
What do you want batteries to do?

Produce current to run some device (i.e. spin a motor) for some period of time.

Per some wiki, take the amperage and divide by 20 (i.e. $\frac{2750}{20}\,\text{mA} = 137.5\ \text{mA}$).

We would then expect this battery could sustain a current of about 137.5 mA for about 20 hours.
I doubt the battery could produce 2750 mA for an entire hour, but it would probably come fairly close (within an hour) of producing the 137.5 mA for 20 hours.

b) The device consumes energy at a rate of $\mathcal{P} = 375\ \text{mW}$. The voltage across the terminals is 1.5 V. Time is $90.0\ \text{min} = 54\underline{0}0\ \text{s}$. We are told internal resistance is negligible.

The current is given by $i = \frac{\mathcal{P}}{\Delta V}$. BUT WAIT! The definition of current is $i = \frac{\Delta Q}{\Delta t}$.

Oh my gosh! Put these together and solve for ΔQ.

$$\Delta Q = \Delta t \cdot i = \frac{\mathcal{P}\,\Delta t}{\Delta V} = 1350\ \text{C}$$

But this is not what the question asked!!! It asked how much charge is left.

$$Q_{remaining} = Q_{init} - \Delta Q = 8550\ \text{C}$$

Again, probably better to think about run times & currents than charge available…

c) Use

$$\mathcal{P} = i\Delta V$$

$$\mathcal{P} = \frac{\Delta Q}{\Delta t}\Delta V$$

$$\Delta t = \frac{\Delta Q}{\mathcal{P}}\Delta V$$

$$\Delta t = \frac{9900\ \text{C}}{375 \times 10^{-3}\ \text{W}} 1.5\ \text{V}$$

$$\Delta t = 39600\ \text{s} = 11\ \text{hrs}$$

In practice the actual run time is probably 80-90% of this number.
One website I read claimed lithium ion batteries tend to be closer to the 90% of this computation.
Maybe this helps you design what size battery to use in some real device someday? Hope so.

26.8

a) A $\mathrm{kW}\cdot\mathrm{hr}$ is a unit of *energy*.

$$1\ \mathrm{kW}\cdot\mathrm{hr} = 10^3\,\frac{\mathrm{J}}{\mathrm{s}}\cdot\mathrm{hr} = 10^3\,\frac{\mathrm{J}}{\mathrm{s}}\cdot(3600\ \mathrm{s}) = 3.6\times10^6\ \mathrm{J} = 3.6\ \mathrm{MJ}$$

b) While charging for 90.0 min = 1.50 hr the charger consumes $4.0\ \mathrm{W}\times1.50\ \mathrm{hr} = 6.0\ \mathrm{W{\cdot}hr}$.
While plugged in, not charging for 22.5 hrs the charger consumes $0.40\ \mathrm{W}\times22.5\ \mathrm{hr} = 9.0\ \mathrm{W{\cdot}hr}$.
Total *daily* energy consumption is $15\ \mathrm{W{\cdot}hr}$.
Monthly energy consumption is $450\ \mathrm{W{\cdot}hr}\approx0.45\ \mathrm{kW{\cdot}hr}$.
Total *monthly* cost is about \$0.054 (or \$0.65 annually).
Notice, however, $\frac{9}{15} = 60\%$ of the cost comes from leaving the charger plugged in all time.
We see one little charger, all by itself, is really not a big deal.
But now, think about perhaps 5 such vampire draws per household (TV, cable box, other chargers, etc).
Be conservative and assume each vampire draw is 5 times smaller than my estimate or 0.4 W.
An overly conservative total monthly cost per household is then given by original estimate.
Times perhaps 100 million households (about 80% of the US households).
Times 12 months.
Divide by 4 (power company charges about 4 times as much to us as it actually costs to create the energy).
Net cost of entire US vampire load is about \$1 million dollars…about 1/50th the annual budget for all of Allan Hancock College in 2016…
Perhaps you have seen those hotels where you have to slide the key in and then the whole room's power comes on. This is a handy way to minimize vampire draws per hotel room. I want one for my house…as long as the fridge isn't on that circuit.

26.9 The units of resistivity are $\Omega \cdot \mathrm{m}$…sorry I forgot them in the problem statement.
a) I will compare the resistivity at 20.0°C (ρ_0) and 3000°C (ρ_{3000}).

$$\rho_{3000} = \rho_0(1 + \alpha\Delta T)$$

$$\frac{\rho_{3000}}{\rho_0} = 1 + \alpha\Delta T$$

$$\alpha = \frac{\frac{\rho_{3000}}{\rho_0} - 1}{\Delta T}$$

$$\boldsymbol{\alpha \approx 5.956 \times 10^{-3}\,\frac{1}{°\mathrm{C}}}$$

b) The bulb is operating at $\mathcal{P} = 60$ W at $\Delta V = 120$ V (being lazy with sig figs here).
Note: if you are worried about AC current (wall socket) versus DC current (batteries), relax.
It turns out this problem works as stated…more in a later chapter on this.
Think: we want to learn about resistance.
Find a version of the power equation involving only power, voltage and resistance!

$$\mathcal{P} = i\Delta V$$

$$\mathcal{P} = \frac{\Delta V^2}{R}$$

$$R = \frac{\Delta V^2}{\mathcal{P}}$$

$$\boldsymbol{R = 240\ \Omega}$$

c) The resistivity can now be computed from

$$R = \frac{\rho L}{A}$$

$$\rho = \frac{AR}{L}$$

$$\rho = \frac{\pi d^2 R}{4L}$$

$$\boldsymbol{\rho = 658.1\ \mathrm{n}\Omega \cdot \mathrm{m}}$$

d) Now use

$$\rho_{operating\ temp} = \rho_0(1 + \alpha\Delta T)$$

$$\Delta T = \frac{\frac{\rho_{operating}}{\rho_0} - 1}{\alpha}$$

$$T_{operating} - 20.0°\mathrm{C} = \frac{\frac{\rho_{operating}}{\rho_0} - 1}{\alpha}$$

I used unrounded answers from previous parts to avoid intermediate rounding errors!

$$T_{operating} = 20.0°\mathrm{C} + \frac{\frac{6.581 \times 10^{-7}\ \Omega \cdot \mathrm{m}}{5.6 \times 10^{-8}\ \Omega \cdot \mathrm{m}} - 1}{5.956 \times 10^{-3}\,\frac{1}{°\mathrm{C}}}$$

$$\boldsymbol{T_{operating} = 1825}\ °\mathrm{C}$$

26.10 NOTE: many of you probably just picked a random point on the curve and plugged it into $R = \frac{\rho L}{A}$.
If you did that, it is impossible to know which point will give the best data. For this type of problem, one with real data, it is often preferable to use the slope to average the data. A solution method using the slope is shown below.
WARNING: the slope method will only work if the plot is linear!

We know $R = \frac{\rho L}{A}$. We notice the plot has vertical axis R and horizontal axis L. I re-write in this form

$$R = \frac{\rho}{A} \cdot L$$

to help me see how it relates to the standard form of the equation for a line

$$y = slope \cdot x + intercept$$

The vertical coordinate of our plot (R) corresponds to y in the standard form.
The horizontal coordinate of our plot (L) corresponds to x in the standard form.
This means the term $\frac{\rho}{A}$ in our equation must correspond to the slope of the graph!

$$slope = \frac{\rho}{A}$$

$$slope = \frac{\rho}{\pi r^2}$$

$$slope = \frac{\rho}{\pi \left(\frac{D}{2}\right)^2}$$

$$slope = \frac{4\rho}{\pi D^2}$$

We were asked to determine the wire diameter…solve this crap for D.

$$D = \sqrt{\frac{4\rho}{\pi \cdot slope}}$$

I estimated the slope of this graph using rise over run to be

$$slope_{estimate} = \frac{rise}{run} = \frac{35\ \mu\Omega}{800\ \text{mm}} = 4.375 \times 10^{-5}\ \frac{\Omega}{\text{m}}$$

When doing ugly computations, it seems easier to do everything in scientific notation. After you find a final result, switch it back to engineering notation with appropriate prefix to make it more palatable for your intended audience.

In this case, I change $\rho = 12.5\ \text{n}\Omega \cdot \text{m}$ to $12.5 \times 10^{-9}\ \Omega \cdot \text{m}$. While not strictly in scientific notation, this is a lot less likely to cause a mistake in computation. It also makes it easier to check units. Plugging in one finds

$$D = \sqrt{\frac{4(12.5 \times 10^{-9}\ \Omega \cdot \text{m})}{\pi \cdot 4.375 \times 10^{-5}\ \frac{\Omega}{\text{m}}}} \approx 19.07\ \text{mm} \approx \text{about}\ \frac{3}{4}\ \text{of an inch}$$

Do these numbers seem reasonable?
The resistivity value is close to that of copper or silver…excellent conductors.
The diameter is almost an inch.
The length is about a meter.
This should give very small resistances…probably on the order of 10's of $\mu\Omega$'s…seems plausible.
Side notes:

- Please keep in mind, real-life data sets are often a little messy; I tried to simulate that in this data set.
- Using a trendline in Excel I found a slope of $4.13 \times 10^{-5}\ \frac{\Omega}{\text{m}}$…within 6% of my estimate.
- According to our theoretical equation $R = \frac{\rho}{A} \cdot L$ we expect the intercept should be zero.
- By forcing the intercept to zero, as one expects in this scenario, the trendline slope was $4.38 \times 10^{-5}\ \frac{\Omega}{\text{m}}$.

26.11 It is important to use unrounded answers in subsequent computations to avoid intermediate rounding errors!

Video showing wire getting hot as voltage is ramped up.

https://www.youtube.com/watch?v=h0Uhx6KY0_0&list=PLBQTyyPKj9WZl89Zhwai-piDCw1DwoPHa&index=10

NOTE: many of you probably just picked a random point on the curve and plugged it into $\Delta V = IR$.
For *this* problem that method works just fine!

For ohmic devices only (devices with a *linear* IV plot) in a lab situation it is often preferable to use $R_{avg} = \frac{1}{slope}$ as it averages the data. I will use this method for part a.

a) We are told $\Delta V = IR$. Rearrange this to isolate I (the vertical coordinate of our plot).

$$I = \frac{1}{R} \cdot \Delta V$$

Notice how this relates to the standard form of

$$y = slope \cdot x + intercept$$

We expect

$$slope = \frac{1}{R_{avg}}$$

I estimated the slope of this graph for $\Delta V < 4.0\text{ V}$ using rise over run.

$$slope_{estimate} = \frac{rise}{run} = \frac{75\text{ mA}}{3.5\text{ V}}$$

$$slope_{estimate} = 2.\underline{1}4 \times 10^{-2}\,\frac{\text{A}}{\text{V}}$$

$$slope_{estimate} = 2.\underline{1}4 \times 10^{-2}\ \Omega^{-1}$$

By looking at our original equation, $\Delta V = IR$, notice $1\ \Omega = 1\frac{\text{V}}{\text{A}}$.
From here we can see

$$R_{avg} = \frac{1}{slope} = 4\underline{6}.7\ \Omega \approx 47\ \Omega$$

b) For a cylindrical wire, $R = \frac{\rho L}{A} = \frac{4\rho L}{\pi D^2}$. Solving for ρ gives

$$\rho = \frac{\pi R D^2}{4L}$$

$$\rho = \frac{\pi(4\underline{6}.7\ \Omega)(325 \times 10^{-6}\text{ m})^2}{4(385 \times 10^{-2}\text{ m})}$$

$$\rho = 1.\underline{0}06 \times 10^{-6}\ \Omega \cdot \text{m} \approx 1\underline{0}06\ \text{n}\Omega \cdot \text{m}$$

What's all that nichrome doing up in here…
Side note on nichrome: two common varieties of nichrome are 80/20 and 70/30.
The resistivity varies wildly depending on which Nichrome variety you get…

c) Notice when $\Delta V = 7.5\text{ V}$ we are now beyond the linear portion of the graph (linear from $\Delta V = 0 \to 4.0\text{ V}$).
The slope trick no longer works!
When $\Delta V = 7.5\text{ V}$ the current is $I = 130\text{ mA}$.

The resistance is $R = \frac{\Delta V}{I} = 5\underline{7}.7\ \Omega \approx 58\ \Omega$.

Problem continues on next page…

d) Resistance versus temperature is given by

$$R(T) = R_{@20°\mathrm{C}}(1 + \alpha\Delta T)$$

WATCH OUT! Here we have defined $\Delta T = T - 20.0°\mathrm{C}$ and T is in °C.

Solving algebraically is the way to go even though we have numbers (some test questions have no numbers).

$$\frac{R(T)}{R_{@20°\mathrm{C}}} = 1 + \alpha\Delta T$$

$$\alpha\Delta T = \frac{R(T)}{R_{@20°\mathrm{C}}} - 1$$

$$\alpha = \frac{\frac{R(T)}{R_{@20°\mathrm{C}}} - 1}{\Delta T}$$

$$\alpha = \frac{\frac{5\underline{7}.7\ \Omega}{4\underline{6}.7\ \Omega} - 1}{123°\mathrm{C} - 20.0°\mathrm{C}}$$

$$\alpha = \frac{1.\underline{2}36 - 1}{103°\mathrm{C}}$$

$$\alpha = \frac{0.\underline{2}36}{103°\mathrm{C}}$$

$$\alpha = 0.00\underline{2}29\frac{1}{°\mathrm{C}} \approx 2 \times 10^{-3}\frac{1}{°\mathrm{C}} = 2 \times 10^{-3}\ \mathrm{K}^{-1}$$

Since both the Celsius and Kelvin temperature scales have the same increment size, this last unit substitution is fair game. As you will learn in a thermo class (or have already learned), when doing problems with **change** in temperature, it is ok to switch between K and °C. This is not a good idea if T, not ΔT, is in your equation.

e) When $\Delta V = 6.25$ V the current is $I = 115$ mA. The resistance is $R = \frac{\Delta V}{I} = 54.\underline{3}\ \Omega$.

$$R(T) = R_{@20°\mathrm{C}}(1 + \alpha\Delta T)$$

$$\frac{R(T)}{R_{@20°\mathrm{C}}} = 1 + \alpha\Delta T$$

$$\alpha\Delta T = \frac{R(T)}{R_{@20°\mathrm{C}}} - 1$$

$$\Delta T = \frac{\frac{R(T)}{R_{@20°\mathrm{C}}} - 1}{\alpha}$$

$$\Delta T = \frac{\frac{54.\underline{3}\ \Omega}{4\underline{6}.7\ \Omega} - 1}{0.00\underline{2}29\frac{1}{°\mathrm{C}}}$$

Notice I used the unrounded result for α to improve the precision of my final result…but ultimately it is only good to 1 sig fig!

$$\Delta T = \underline{7}1.5\ °\mathrm{C}$$

THIS IS NOT THE ANSWER!!!!

I asked for temperature…not *change* in temperature. Use $\Delta T = T - 20.0°\mathrm{C}$

$$\boldsymbol{T = \underline{9}1.5}\ °\mathrm{C}$$

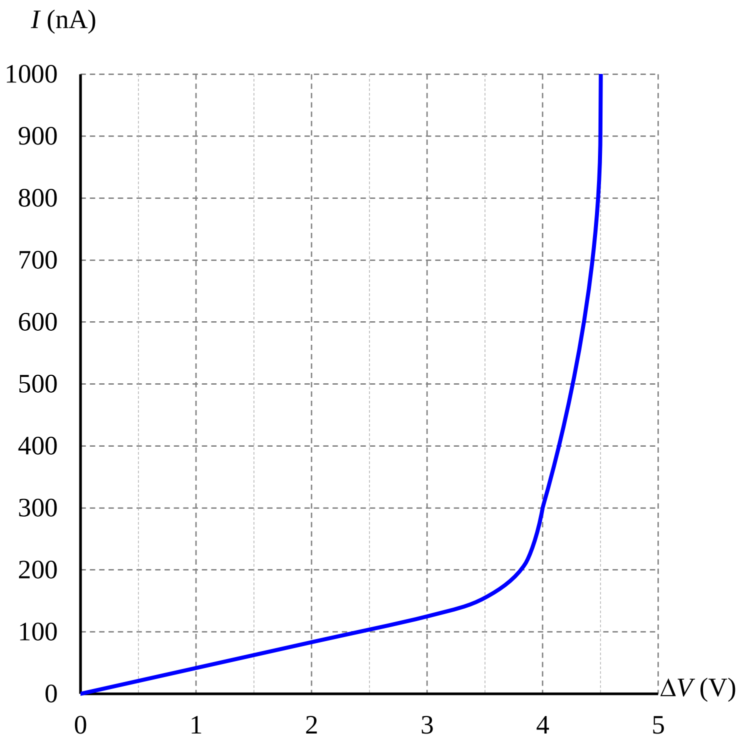
f) The slope in that region is about

$$slope_{estimate} = \frac{rise}{run} = 1.\underline{2}7 \times 10^{-2}\frac{\mathrm{A}}{\mathrm{V}} = 1.\underline{2}7 \times 10^{-2}\ \Omega^{-1}$$

This gives

$$R_{diff} = \frac{1}{slope} \approx 79\ \Omega$$

26.12 Germanium has resistivity 0.46…no × 10 at all!!!
That is about half a million times larger than Nichrome's resistivity.
We expect resistances to be about half a million times smaller.
Furthermore, the temperature coefficient for germanium is $-48 \times 10^{-3}\,\frac{1}{°\mathrm{C}}$.
That is about 120 times larger than Nichrome's temperature coefficient of resistivity AND NEGATIVE.
This means we expect resistance to change more rapidly AND decrease (instead of increase).
If resistance decreases we expect the average slope to increase.
We expect the increase in slope to be 120 times more dramatic the change observed for nichrome.
In the plot below, notice I'm using NANOamps now as compared to milliamps for nichrome.
DISCLAIMER: I have no clue if our fictional germanium wire should experience comparable temperature changes to the nichrome wire…perhaps it radiates heat much less effectively and would experience much different temperature changes.



26.13 Let subscript 1 represent carbon & subscript 2 represent Nichrome.
We are told

$$R_1 + R_2 = 10.0\ \Omega$$

We are told the wires resistance does NOT change with temperature.
If you are clever, you might realize this implies the increase of the Nichrome segment ($\alpha_2 > 0$) must be exactly offset by the the decrease in the carbon segment ($\alpha_1 < 0$).
In equation form

$$\Delta R_2 = -\Delta R_1$$
$$R_{0\ for\ 2}\alpha_2\Delta T_2 = -R_{0\ for\ 1}\alpha_1\Delta T_1$$

In this case, the wires are in thermal contact. We assume they experience identical temperature changes.

$$R_{0\ for\ 2}\alpha_2 = -R_{0\ for\ 1}\alpha_1$$
$$R_{0\ for\ 2} = -\frac{\alpha_1}{\alpha_2}R_{0\ for\ 1}$$

We now have two equations and two unknowns!

$$R_1 + R_2 = 10.0\ \Omega$$
$$R_{0\ for\ 1} - \frac{\alpha_1}{\alpha_2}R_{0\ for\ 1} = 10.0\ \Omega$$
$$R_{0\ for\ 1}\left(1 - \frac{\alpha_1}{\alpha_2}\right) = 10.0\ \Omega$$
$$R_{0\ for\ 1} = \frac{10.0\ \Omega}{1 - \frac{\alpha_1}{\alpha_2}}$$
$$\frac{\rho_{0\ for\ 1}L_1}{A_1} = \frac{10.0\ \Omega}{1 - \frac{\alpha_1}{\alpha_2}}$$
$$L_1 = \frac{A}{\rho_{0\ for\ 1}} \cdot \frac{10.0\ \Omega}{1 - \frac{\alpha_1}{\alpha_2}}$$
$$\boldsymbol{L_1 = 1.596}\ \mathbf{m}$$

In a similar fashion, one also finds $L_2 = 63.5$ m.

BONUS: These distances are seriously unwieldy.
If we want them to be 100 times shorter, the area must be reduced by 100 times as well.
This implies the radius need only be reduced by a factor of 10.
We'd need a wire that is 0.200 mm instead of 2.00 mm.
This reduces lengths to about 1.6 cm and 63.5 cm…
This is much more manageable…but the wire is more easily snapped.

27.14

a) We are told total *resistance* is R and given

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$

We are also told the carbon core and nichrome shell have equal resistance ($R_1 = R_2$). Plugging in we find

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_1}$$
$$\frac{1}{R} = \frac{2}{R_1}$$
$$R_1 = 2R$$

Same for core and shell.

b) To determine diameter of the core

$$R_{core} = \frac{\rho_{core} L}{A_{core}} = \frac{4\rho_{core} L}{\pi D^2}$$

Solving for diameter gives

$$\boldsymbol{D = \sqrt{\frac{4\rho_{core} L}{\pi R_{core}}} = \sqrt{\frac{2\rho_1 L}{\pi R}}}$$

c) Now use

$$R_{thin\ shell} = \frac{\rho_{shell} L}{A_{shell}}$$
$$R_{thin\ shell} = \frac{\rho_{shell} L}{(circumference) \times (shell\ thickness)}$$
$$R_{thin\ shell} = \frac{\rho_{shell} L}{(\pi D) \times (t)}$$
$$t = \frac{\rho_{shell} L}{\pi D R_{thin\ shell}}$$
$$t = \frac{\rho_2 L}{\pi \left(\sqrt{\frac{2\rho_1 L}{\pi R}}\right)(2R)}$$

Bring all crap in denominator into radical.

$$t = \frac{\rho_2 L}{\sqrt{8R\pi\rho_1 L}}$$

Might as well bring it all in I guess…

$$t = \sqrt{\frac{\rho_2^2 L}{8R\pi\rho_1}}$$

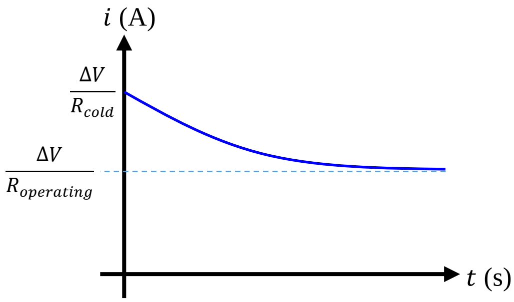
This is getting outrageous…better check the units

$$[t] = \sqrt{\frac{[\rho_2]^2[L]}{[R][\rho_1]}} = \sqrt{\frac{(\Omega \cdot \mathrm{m})^2(\mathrm{m})}{(\Omega)(\Omega \cdot \mathrm{m})}} = \sqrt{\frac{\Omega^2 \cdot \mathrm{m}^3}{\Omega^2 \cdot \mathrm{m}}} = \mathrm{m}$$

Looks good as far as units go.

d) As it heats up, the carbon core resistance decreases while nichrome resistance increases. In parallel, we will learn the net resistance is always less than either resistor by itself. This means the resistance of this unusual resistor actually goes down with temperature. If you didn't see this, rest assured you would had you done the homework from the next chapter.

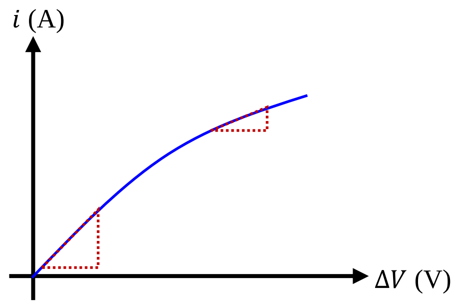
26.15 Video shows a Nichrome wire getting hot and burning…
https://www.youtube.com/watch?v=h0Uhx6KY0_0&list=PLBQTyyPKj9WZl89Zhwai-piDCw1DwoPHa&index=10
The filament is made of metal (i.e. tungsten).
We know as current flows in the wire, the metal temperature should increase.
As temperature increases, we expect resistance to increase (and thus current decreases).



Note: one web resource I found indicated temperatures reached 90% of operating temps in *about* 0.1 seconds for some random 10 W and 100 W bulbs. These time frames seem plausible enough to me.

26.16 Video shows a Nichrome wire getting hot and burning…
https://www.youtube.com/watch?v=h0Uhx6KY0_0&list=PLBQTyyPKj9WZl89Zhwai-piDCw1DwoPHa&index=10

As voltage is increased, current should increase.
Temperature should also increase.
This implies resistance should also increase.
This implies resistance at high voltage should be slightly larger than resistance at low voltage.
This implies changing voltage from 0 to 1 V increases current slightly more than increasing voltage from 4 to 5 V.



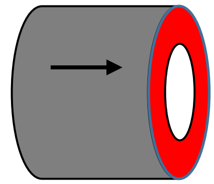
Notice the two red triangles in the figure have the same base (same ΔV).
At high voltage, however, the triangle is shorter (less increase in current).
Note: this figure is almost certainly not to scale.
In my experience, the curve is *almost* a straight line.
Also, the wires I've used tend to melt fairly soon after you notice any significant deviation from a straight line.

26.17

a) $14.36\ \mathrm{m\Omega}$
b) 626.7 A…
   That's a LOT of current.
   This can occur if you connect a car battery to a small gauge (large radius) wire with no other resistance.
c) $19.96\ \mathrm{m\Omega}$
d) 450.8 A
e) 538.8 A
f) 4.849 kW
g) 5.301×10^{-5} kg
h) $This\ part\ not\ on\ a\ \ PHYS\ 163\ test$ … 4.771 J
i) Use $\mathcal{P} = \frac{\Delta E}{\Delta t} \rightarrow \Delta t = \frac{\Delta E}{\mathcal{P}} \approx 9.84 \times 10^{-4}$ s ≈ 1 msec.

Please note: this is an over-simplified estimate.
In practice, much of the power delivered to the wire is radiated away.
This means the time to reach temperature must be LONGER than what we estimate here.
That said, it gives us a reasonable estimate for the minimum time to reach temperature.
The colder the surroundings, the more one must increase this time estimate.

26.18



a) For case one, there is no need to perform calculus.

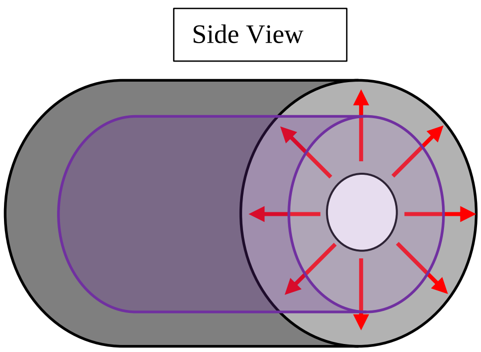
Current is uniformly distributed throughout the red-cross-sectional area

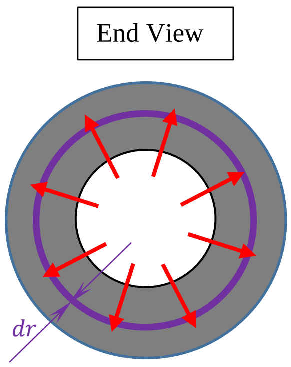
$$A = \pi R_{outer}^2 - \pi R_{inner}^2$$
$$A = \pi b^2 - \pi a^2$$
$$A = \pi(b^2 - a^2)$$

Resistance is thus

$$R = \frac{\rho L}{A}$$
$$\boldsymbol{R = \frac{\rho L}{\pi(b^2 - a^2)}}$$

b) The figure below hopefully helps…





Notice the cross-sectional area is now the sidewall of purple cylinder (current flows through the sidewall).

The area is now $A = 2\pi r L$.

The thickness of the cylindrical shell is dr.

Because the dimensions of the sidewall area change, we must use

$$R = \int_{inner\ radius}^{outer\ radius} \frac{\rho\, ds}{A}$$
$$R = \int_a^b \frac{\rho\, dr}{2\pi r L}$$
$$\boldsymbol{R = \frac{\rho}{2\pi L} \ln\left(\frac{b}{a}\right)}$$

c) Setting the two resistances equal gives

$$\frac{\boldsymbol{\rho}}{\boldsymbol{2\pi L}} \boldsymbol{\ln \frac{b}{a}} = \frac{\rho L}{\pi(b^2 - a^2)}$$
$$L^2 = \frac{\ln \frac{b}{a}}{2} (b^2 - a^2)$$
$$\boldsymbol{L = \sqrt{\frac{\ln \frac{b}{a}}{2}} \sqrt{b^2 - a^2}}$$

26.19 The figure at right hopefully helps…

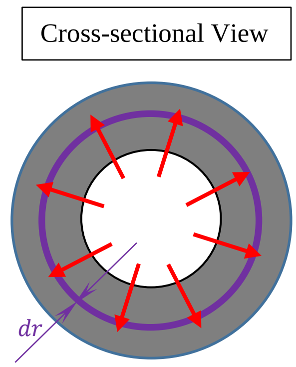
In this manner current can flow from the inner surface of the cylindrical shell

Notice the cross-sectional area is now the spherical shell indicated by the purple circle.

The area is now $A = 4\pi r^2$.

The thickness of the shell is dr.

Because the dimensions of the sidewall area change, we must use



$$R = \int_{inner\ radius}^{outer\ radius} \frac{\rho\, ds}{A}$$

$$R = \int_a^b \frac{\rho\, dr}{4\pi r^2}$$

$$R = -\frac{\rho}{4\pi r}\bigg|_a^b$$

If I flip the limits I can flip the sign…

$$R = \frac{\rho}{4\pi r}\bigg|_b^a$$

$$R = \frac{\rho}{4\pi a} - \frac{\rho}{4\pi b}$$

$$\boldsymbol{R = \frac{\rho(b-a)}{4\pi ab}}$$

Checks: units work out.

Also, we were told $b > a$ so this is a positive number (as resistances should be).

Finally: if $b \approx a$, we'd be have a very thin shell which should have nearly zero resistance.

Our formula goes to zero as $b \to a$ as we expect. Looks good.

Note: in practice one would connect to the inner wall of the spherical shell by drilling a small hole.

An insulated wire then is passed through the hole to the interior wall.

The resistive medium between the two surfaces might actually be air (instead of a solid metal shell).

Such a device reminds me a bit of a device used in measuring radioactivity.

I suspect it is much simpler to make the cylindrical version…but this spherical version could be made in real life.

26.20

a) Between the front and back faces, cross-sectional area remains constant.
Non need for calculus.

$$R = \frac{\rho L}{A}$$

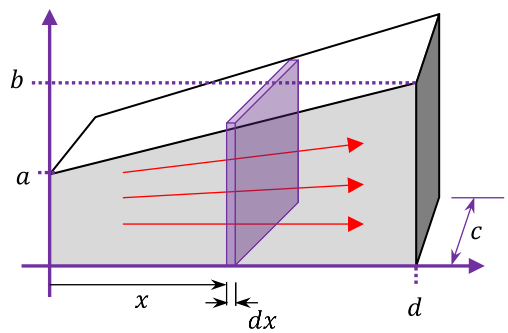
Here the cross-sectional area is that of the trapezoidal face.

$$A = \frac{1}{2}(a+b)d$$

The length of the resistor is the distance between the front and back faces…in this case c.

$$\boldsymbol{R = \frac{2\rho c}{(a+b)d}}$$

b) When the left and right faces are used, the cross-section through which current flows changes.
My attempt to draw this is shown at right.



Current is indicated by the red arrows.
The purple slab is supposed to be a cross-section at some arbitrary distance x for the left face.

Note: the cross-section has constant width c into the page while the thickness of the slab is dx.
The height of the slab changes linearly.
The linear function is

$$y = slope \cdot x + intercept$$
$$y = \frac{rise}{run} \cdot x + intercept$$
$$y = \frac{b-a}{d}x + a$$

Anytime I write a line function, I always try to check it works properly by plugging in the two endpoints.
If I plug in $x = 0$ I get $y = a$.
If I plug in $x = d$ I get $y = b$.
Looks good.
The integral looks like

$$R = \int_0^d \frac{\rho\, dx}{c\left(\frac{b-a}{d}x + a\right)}$$

TIP: at this point I usually do a seemingly odd thing.
Instead of doing a U sub, I simply rewrite this integral in terms of slab height y!
Because $y = \frac{b-a}{d}x + a$ we know $\frac{dy}{dx} = \frac{b-a}{d}$.
In turn this tells us $dx = \frac{d}{b-a} \cdot dy$.
Also, going vertically, the limits are from $a \to b$!
The integral becomes

$$R = \int_a^b \frac{\rho \frac{d}{b-a} \cdot dy}{cy}$$

$$\boldsymbol{R = \frac{\rho d}{c(b-a)} \ln \frac{b}{a}}$$

I've seen this trick work for a huge number of these resistance integrals…or just U-sub it…you get the same thing.

26.21 Determining the function (in part a) is probably more useful to your career than as exam practice.
a) We are told the height function varies parabolically.

$$y = k_0 + k_1 x + k_2 x^2$$

Here $k_0, k_1, \& \ k_2$ are constants.
We are told the minimum occurs at the left face.
If you are clever, you might recognize by symmetry $k_1 = 0$ and thus $k_0 = a$ and from there find $k_2 = \frac{b-a}{d^2}$.
If you are not clever, grind it out.
We know the following:

i. $y = a$ when $x = 0$
ii. $y = b$ when $x = d$
iii. $y' = 0$ when $x = 0$

From each bullet, you get an equation.
There should be three equations to figure out the three unknown constants.

Using bullet i: $a = k_0 + k_1(0) + k_2(0)^2$ which gives $k_0 = a$
Using bullet iii: $0 = k_1 + k_2(0)^2$ which gives $k_1 = 0$
Using bullet ii: $b = a + k_2 d^2$…which gives $k_2 = \frac{b-a}{d^2}$

Notice one finds

$$y = a + \frac{b-a}{d^2} x^2$$

It would be foolish to proceed without checking this formula works for both $x = 0$ & $x = d$ (it checks out).
Part b starts on the next page…

b) The integral becomes

$$R = \int_0^d \frac{\rho\, dx}{yc} = \int_0^d \frac{\rho\, dx}{\left(a + \frac{b-a}{d^2}x^2\right)c}$$

In real life, you probably go straight to an online integrator at this point.
Note: if we persist with our trick (switch from an x integral to a y integral) I believe it is also pretty easy to solve with a table. As is it is cumbersome but probably doable with a table as well.

Sub out dx using

$$\frac{dy}{dx} = 2\frac{b-a}{d^2}x$$

$$\frac{dy}{2\frac{b-a}{d^2}x} = dx$$

Because $y = a + \frac{b-a}{d^2}x^2$ we can invert to find $x = \left(\frac{d^2}{b-a}\right)^{\frac{1}{2}}(y-a)^{1/2}$

This gives

$$R = \frac{\rho d}{2c\sqrt{b-a}} \int_a^b \frac{dy}{y(y-a)^{1/2}}$$

Yeesh…table time…(notice the limits now correspond to the y-axis limits).
Maybe straight up is better after all…

$$R = \int_0^d \frac{\rho\, dx}{\left(a + \frac{b-a}{d^2}x^2\right)c}$$

$$R = \int_0^d \frac{\rho\, dx}{\frac{b-a}{d^2}\left(\frac{d^2}{b-a}a + x^2\right)c}$$

$$R = \frac{d^2\rho}{c(b-a)} \int_0^d \frac{dx}{\left(x^2 + \frac{d^2}{b-a}a\right)}$$

Now let $crap^2 = \frac{d^2}{b-a}a$

$$R = \frac{d^2\rho}{c(b-a)} \int_0^d \frac{dx}{(x^2 + crap^2)}$$

Using a table gives

$$R = \frac{d^2\rho}{c(b-a)} \left[\frac{1}{crap}\tan^{-1}\left(\frac{x}{crap}\right)\right]_0^d$$

Since $\tan^{-1} 0 = 0$

$$R = \frac{d^2\rho}{c(b-a)} \cdot \frac{1}{crap}\tan^{-1}\left(\frac{d}{crap}\right)$$

$$R = \frac{d^2\rho}{c(b-a)} \cdot \frac{1}{\sqrt{\frac{d^2}{b-a}a}}\tan^{-1}\left(\frac{d}{\sqrt{\frac{d^2}{b-a}a}}\right)$$

$$\boldsymbol{R = \frac{d\rho}{c\sqrt{a(b-a)}}\tan^{-1}\left(\sqrt{\frac{b-a}{a}}\right) = \frac{d\rho}{ca\sqrt{\left(\frac{b}{a}-1\right)}}\tan^{-1}\left(\sqrt{\frac{b}{a}-1}\right)}$$

The second form might help when checking units.

Part c) After checking the units, I worried about what happens as $a \to 0$.

The resistor becomes infinitely skinny and resistance should trend to infinity.

I used an online program to verify the limits made sense for both $a \to 0$ and $b \to a$.

Think: what should the resistance be if $b = a$? It should be very simple to compute without calculus…

Part d) The cross sectional area is constant (which *usually* implies no calculus is necessary).

HOWEVER, we must use calculus to determine this particular cross-sectional area!

I guess that's a trick question, right?

Get the area using $A = \int dA = \int y\, dx$.

Then shove into $R = \frac{\rho L}{A}$ where $L = c$ and $A = \int y\, dx$ is the result of the integral.

Part e) If resistivity is non-uniform, simply do

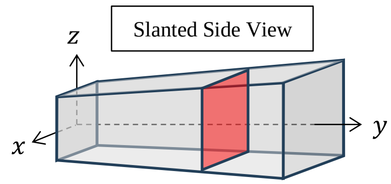
$$R = \int_0^d \frac{\rho\, dx}{c\left(\frac{b-a}{d^2}x^2 + a\right)}$$

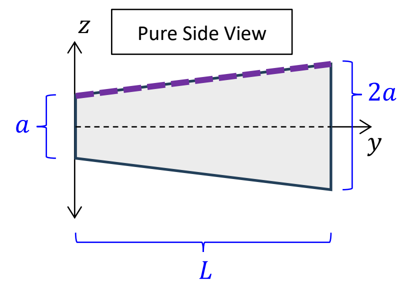
At this point, one plugs in the equation for resistivity determined from the problem statement!

26.21½ Since cross-sectional area varies between the two faces, we must integrate to determine resistance.

$$R = \int_i^f \frac{\rho\, ds}{A}$$

In this coordinate system, current travelling left to right indicates current travels along the y-axis. Therefore $ds = dy$.





The cross-sectional area is square but the side length of the square grows linearly. Notice the dashed purple line in the lower figure along the top edge of the prism. Hopefully it makes sense why I would choose to write the equation

$$z = slope \cdot y + intercept$$

Notice I used z & y instead of y & x because of the orientation of the coordinate system in the lower figure at right.

Notice $intercept = \frac{a}{2}$ while

$$slope = \frac{rise}{run} = \frac{a - \frac{a}{2}}{L} = \frac{a}{2L}$$

Plugging in gives

$$z = \frac{a}{2L} y + \frac{a}{2}$$

Notice the area of an arbitrary face at some random location in the prism is

$$A = (2z)^2 = \left(2\left(\frac{a}{2L} \cdot y + \frac{a}{2}\right)\right)^2 = \left(\frac{a}{L} \cdot y + a\right)^2$$

Think: you should *not* use area a^2 (or $4a^2$)…the area of the left (or right) face is not representative of an arbitrary slice of the prism. From here, plug everything in:

$$R = \int_0^L \frac{\rho\, dy}{\left(\frac{a}{L} y + a\right)^2}$$

Notice the units correspond to distance along the y-axis (along the direction of current flow).
I notice we could use

$$u = \frac{a}{L} y + a \quad \to \quad du = \frac{a}{L} dy \quad \to \quad dy = \frac{L}{a} du$$

I also switch the limits by noting $u = a$ when $y = 0$ and $u = 2a$ when $y = L$.

$$R = \int_a^{2a} \frac{\rho \frac{L}{a} du}{u^2}$$

$$R = \frac{\rho L}{a} \left[-\frac{1}{u}\right]_a^{2a}$$

$$\boldsymbol{R = \frac{\rho L}{2a^2}}$$

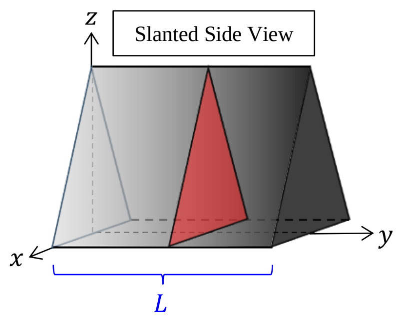
26.21¾

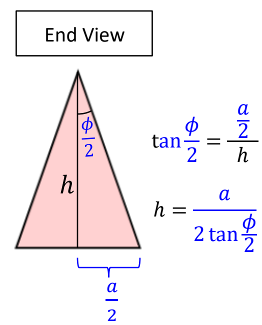
Part a: Determine the units of the constant

$$[\rho] = [\beta][y^2] \quad \rightarrow \quad [\beta] = \frac{[\rho]}{[y^2]} = \frac{\Omega \cdot \text{m}}{\text{m}^2} = \frac{\boldsymbol{\Omega}}{\textbf{m}}$$

Part b: In this problem the cross-sectional area is constant but resistivity is non-uniform. We must integrate to determine resistance.

$$R = \int_i^f \frac{\rho\, ds}{A}$$





Here I will pull out the constants early. In particular, since area is constant we can pull that out right away this time.

$$R = \frac{1}{A}\int_0^L \rho\, dy$$

I suppose we should use the given parameters to determine the area (see end view figure above).
I will compute it now but not plug it in until after the integral to reduce time spent writing it repeatedly.

$$A = \frac{1}{2} base \times height = \frac{1}{2} ah = \frac{a^2}{4\tan\frac{\phi}{2}}$$

$$R = \frac{1}{A}\int_0^L \beta y^2\, dy$$

$$R = \frac{1}{A}\beta \int_0^L y^2\, dy$$

$$R = \frac{1}{A} \cdot \frac{\beta L^3}{3}$$

Think: always check the zero limit. In this case it drops out.

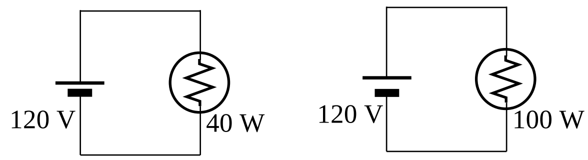
$$R = \frac{4\tan\frac{\phi}{2}}{a^2} \cdot \frac{\beta L^3}{3}$$

Before plugging numbers into a calculator to make it look pretty, I check the units (looks good).

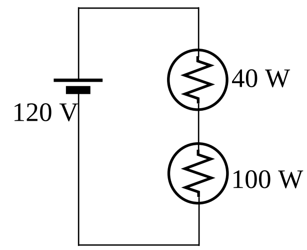
$$\boldsymbol{R = 0.485 \frac{\beta L^3}{a^2}}$$

26.22 & **26.23** **I'd watch this video first to get a feeling for these two problems.**
https://www.youtube.com/watch?v=UM8Ow5w8Cs8&list=PLBQTyyPKj9WbIWdZMnPQeUwedY2u3lrxt
We will learn more about the differences between alternating current (AC) & direct current (DC) later. Fortunately, resistor only circuits used with AC power (plugged into a wall socket) have computations nearly identical to DC circuits (plugged into a battery). With old incandescent light bulbs, the circuit is essentially a long skinny wire inside a piece of glass. These bulbs can be modeled very well as simple resistors.

In a standard household circuit, the entire 120 V potential difference is applied to each bulb (shown in the figure at right). Note: in this simple case, the power rating directly corresponds to brightness.



In question **26.22**, we are asked to wire them in series as shown in the figure at right. Notice something is very different now! Because the bulbs are in series, neither bulb gets the full 120 V potential difference of the source! As a result, the ratings for wattage on each bulb are no longer accurate!



To understand the *lower* figure, first use the *upper* figure to figure out the resistance of each bulb.
We know two equations:

Power	Ohm's Law
$\mathcal{P} = i\Delta V$	$\Delta V = iR$

In the upper figure we know *power* & *voltage* and want to learn about *resistance*.
It makes sense to use these two equations to eliminate *current*.

$$\mathcal{P} = i\Delta V = \left(\frac{\Delta V}{R}\right)\Delta V = \frac{\Delta V^2}{R}$$

$$R = \frac{\Delta V^2}{\mathcal{P}}$$

In the upper figure, we see the two bulbs have the same voltage applied (120 V each).
The resistances are $R_{40\text{ W}} = 360\ \Omega$ & $R_{100\text{ W}} = 144\ \Omega$.

In the lower figure, the two resistors ins series have total resistance $R_{total} = R_{40\text{ W}} + R_{100\text{ W}} = 504\ \Omega$.
Since we know total resistance and voltage, we can determine current.

$$i = \frac{\Delta V}{R_{total}} = 238.1\text{ mA}$$

Since these resistors are in series, we know they share the same current.
Since we know *resistance* & *current*, we can determine *power* (which corresponds to brightness).
This time it makes sense to eliminate *voltage* from our two equations.

$$\mathcal{P} = i\Delta V = i(iR) = i^2R$$

Now we can figure out the power deliverd to each bulb in the series circuit (lower figure).

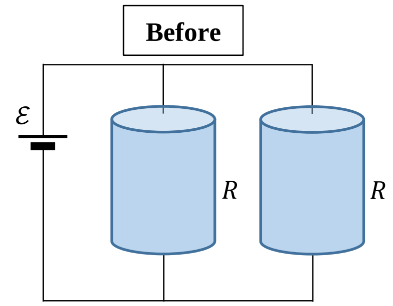
$$\mathcal{P}_{40\text{ W}} = (238.1\text{ mA})^2(360\ \Omega) = 20.4\text{ W}$$

$$\mathcal{P}_{100\text{ W}} = (238.1\text{ mA})^2(144\ \Omega) = 8.16\text{ W}$$

Notice the bulb labeled "40 W" is delivered about 20 W while the bulb labeled "100 W" is delivered about 8 W. The 40 W bulb is brighter when the two bulbs are placed in series!!!!!

Note: in real life, changing the circuit to a series circuit (lower figure) also affects operating temperature. When temperature is factored in, the smaller bulb gets even brighter! This problem is supposed to motivate you to transition into the next chapter where resistor circuits are discussed in greater detail.

26.24 Note: I cover series versus parallel rules for resistors in the next chapter in great detail.
Now to the solution solution (ha.):



In the first case we know

$$\mathcal{P} = i\Delta V$$

$$\mathcal{P} = \left(\frac{\Delta V}{R}\right)\Delta V$$

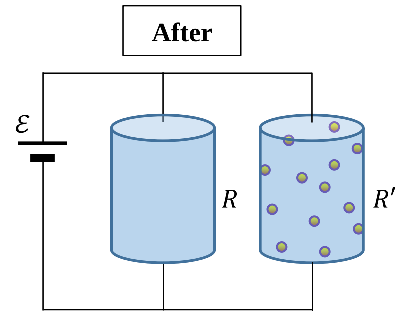
$$\mathcal{P} = \frac{\Delta V^2}{R}$$

We don't know source voltage but we can give that source voltage a name.
Because the second circuit has the same unknown source voltage, it should drop out in a ratio!
Each resistor in *parallel* with the source has the same potential difference as the source.

$$\mathcal{P} = \frac{\mathcal{E}^2}{R}$$

Total power delivered to both cylinders is thus

$$\mathcal{P}_{total\ before} = 2\frac{\mathcal{E}^2}{R}$$



In the second case, *resistivity* in one resistor has dropped by 25%.
Recall $R = \frac{\rho L}{A}$…*resistance* is directly proportional to *resistivity*.
This implies *resistance* also drops by 25%.
Dropping by 25% is the same as saying $R' = 0.75R$ (R' is75% of original value).
The same power equation applies to the modified resistor.

$$\mathcal{P}' = \frac{\mathcal{E}^2}{R'} = \frac{\mathcal{E}^2}{0.75R} = 1.333\frac{\mathcal{E}^2}{R}$$

Total power in the second case is thus

$$\mathcal{P}_{total\ after} = \frac{\mathcal{E}^2}{R} + 1.333\frac{\mathcal{E}^2}{R} = 2.333\frac{\mathcal{E}^2}{R}$$

We are asked to find the ratio of power after to power before:

$$\frac{\mathcal{P}'}{\mathcal{P}} = \frac{2.333\frac{\mathcal{E}^2}{R}}{2\frac{\mathcal{E}^2}{R}}$$

$$\boldsymbol{\frac{\mathcal{P}'}{\mathcal{P}} = 1.167}$$

Dropping the resistivity of a single cylinder increases power delivered by about 17%.

In the real world, there are occasions where resistivity measurements are used in practical settings.
While this particular problem seems contrived, it relates to these practical techniques.
One example: geologists can use resistivity measurements to determine the subsurface structure of a plot of land before anyone wastes time and money digging.
Another example: use resistivity (or conductivity) measurements to estimate Total Dissolved Solids (TDS) in water.
Many other applications can be found in electrical engineering.