Ch8

8.1 Try it with the sims. Problem **8.3** handles some of the parts as well. Regarding the challenge, I found the block loses the same fraction (percentage) of height with each pass.

8.2 Try the sims. Problem **8.4** handles some of the parts as well.

8.3

a) Starting with $E_i + W_{non-con} = E_f$. There is negligible friction. The normal force always points perpendicular to the direction of travel. The only other force is gravity. In this case $W_{non-con} = 0$ so energy is conserved. This gives

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$$

$$mgh = \frac{1}{2}mv_f^2$$

Mass drops and we find $v_{max} = \sqrt{2gh}$.

b) Again using conservation of energy

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$$

$$mgh = \frac{1}{2}m\left(\frac{v_{max}}{2}\right)^2 + mgh_f$$

$$gh = \frac{1}{2}\left(\frac{\sqrt{2gh}}{2}\right)^2 + gh_f$$

$$gh = \frac{gh}{4} + gh_f$$

$$h_f = \frac{3}{4}h$$

You get half the speed in the first ¼ of the fall!

Another style: We know that after a block has *fallen* distance h the speed is $v = \sqrt{2gh}$. Comparing any two fall distances with a ratio gives

$$\frac{v'}{v} = \frac{\sqrt{2gh'}}{\sqrt{2gh}} = \sqrt{\frac{h'}{h}}$$

$$h' = h\left(\frac{v'}{v}\right)^2$$

If we want half the speed we get

$$h' = h\left(\frac{v/2}{v}\right)^2 = \frac{h}{4}$$

The distance *above the ground* is initial height minus distance fallen which gives the same result as before.

c) $\frac{v'}{v} = \frac{\sqrt{2gh'}}{\sqrt{2gh}} = \sqrt{\frac{h'}{h}} = \sqrt{\frac{1}{2}} \approx 0.7071$. Another way to compare them is to say the speed is 71% of max.

8.4

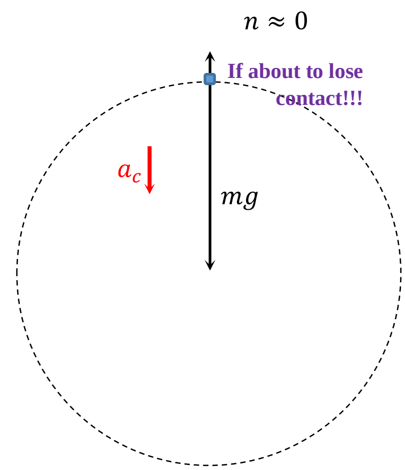
a) Again $W_{non-con} = 0$. Don't get lazy; try to always check this just in case. Starting from rest there is no initial *kinetic* energy. The energy equation comparing the initial position to any other point gives

$$mgh = \frac{1}{2}mv_f^2 + mgh_f$$
$$2gh = v_f^2 + 2gh_f$$
$$v_f = \sqrt{2g(h - h_f)}$$

Point	$\boldsymbol{h_f}$	$\boldsymbol{v}$
A	0	$\sqrt{2gh}$
B	R	$\sqrt{2g(h-R)}$
C	$2R$	$\sqrt{2g(h-2R)}$

b) Just like the swing a bucket of water problem (**6.55**) the normal force at the top must be greater than or equal to zero. If the block is going very slowly, it is about to lose contact with the loop at the top. We can learn about the *minimum* speed to make it through the loop by setting $n = 0$ (at the top).

Setting the normal force to zero gives $v_{min} = \sqrt{Rg}$. Setting this result equal to our result for v_C from part a gives $h_{min} = 2.5R$.



8.5

a) The speed at the top of the loop in each case is the same because, in each case, the riders experience the same change in height.

b) The acceleration at max height is purely centripetal ($a_c = \frac{v^2}{r}$ where r = radius of curvature). The radius of curvature is largest in **B** and smallest in **C**. We find

$$a_\mathbf{C} > a_\mathbf{A} > a_\mathbf{B}$$

c) If you do an FBD the normal force is given by $n = ma_c - mg$. We see

$$n_\mathbf{C} > n_\mathbf{A} > n_\mathbf{B}$$

8.6 No air resistance so $W_{non-con} = 0$. Gravity is only force on ball after launch. Using energy we find

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$$
$$v^2 + 2gh = v_f^2$$
$$v_f = \sqrt{v^2 + 2gh}$$

Isn't that much easier than kinematics?!?!? Of course, if you need time or range kinematics is more useful.

Kinematics style: The impact speed is given by $v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$. We know $v_{fx} = v_{ix} = v\cos\theta$.

From the v^2 equation we know $v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y = (v\sin\theta)^2 + 2(-g)(-h) = v^2\sin^2\theta + 2gh$

Plugging in we get $v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{v^2\cos^2\theta + v^2\sin^2\theta + 2gh} = v_f = \sqrt{v^2 + 2gh}$

8.7

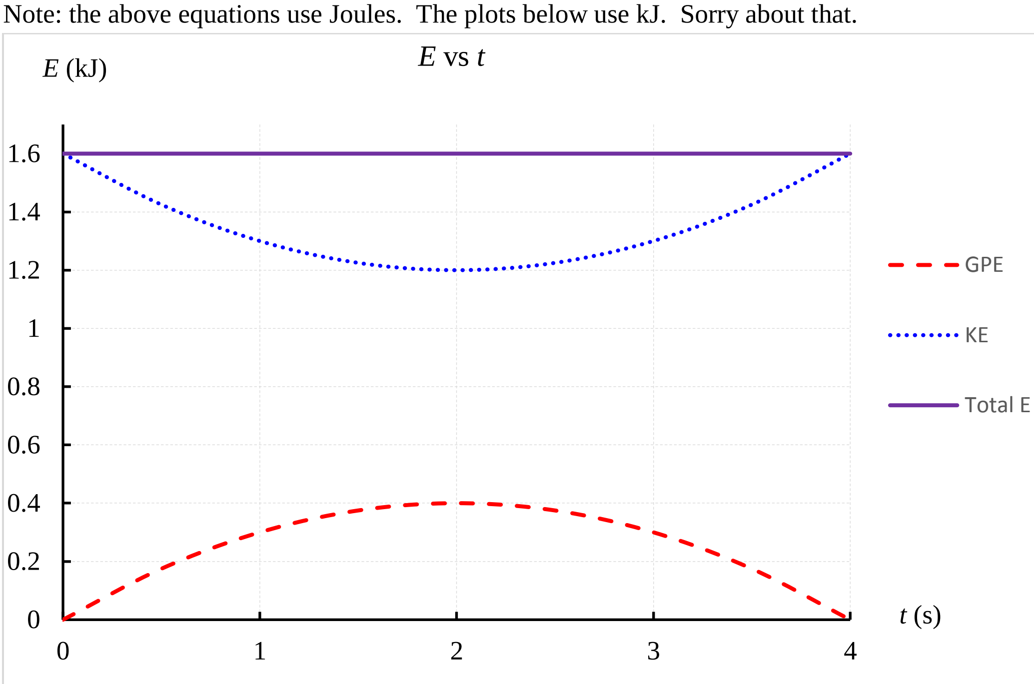
a) See plots below. Did you remember that kinetic energy is non-zero at max height?

b) I will leave off units for brevity. $v = \sqrt{v_x^2 + v_y^2} = \sqrt{100t^2 - 400t + 1600}$ and $y = -5t^2 + 20t$

c) $U_G(t) = mgy = -100t^2 + 400t$ and $K(t) = \frac{1}{2}mv^2 = 100t^2 - 400t + 1600$

Total energy is the sum of these two $E_{total}(t) = K(t) + U_G(t) = 1600$.

Note: the above equations use Joules. The plots below use kJ. Sorry about that.



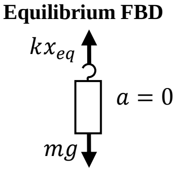
8.8 I did a web search for "masses and springs interactive"
I found a simulation and place d a 100 g mass onto a spring.
I let the spring come to rest (by hitting the stop button).
I used this information to determine the spring constant using the FBD shown at right.

a) If you look carefully, the 100 gram stretches the spring by 9.8 cm not 10 cm!

b) Using the FBD at right we see $kx_{eq} = mg$. I want to point out that I found $k \approx 10\,\frac{\text{N}}{\text{m}}$. Assuming we have three sig figs on the masses (common in lab), we should get about two sig figs for k.
Note: kx_{eq} is a positive quantity. It is the magnitude of the force. The direction of the force will handle the sign. I assume the spring *force* is $\vec{F} = -k\vec{x}$ while the *magnitude* is $F = kx$.

c) The equilibrium position for the 250 g mass is 24.5 cm.

Equilibrium FBD



8.9

a) Do the sim…Only energy form is GPE.

b) For a vertical spring system GPE, KE, and SPE are all present at equilibrium!

c) At max stretch we have GPE and SPE. The reason there is still GPE in the simulation is they set the reference level as the ground…not at the lowest point in the oscillation. Typically we would set the lowest point of the oscillation as our reference level and then GPE would also be zero at the bottom.

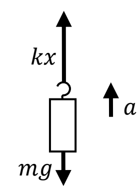
d) For mass-spring system released from rest at unstretched position the max stretch is $2x_{eq}$!

e) Only $\vec{v} = 0$. Think: when you throw a ball vertically the velocity is zero at max height but not the acceleration. An FBD is shown for the max stretch at right.

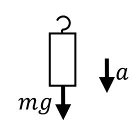
f) At equilibrium $\vec{a} = 0$ and $\vec{F}_{net} = 0$. The velocity is actually at a maximum. FBD was shown in Experiment 1 solution.

g) Both $\vec{v} = 0$ and $\vec{F}_{spring} = 0$. FBD shown at right.

Max Stretch FBD



Unstretched FBD



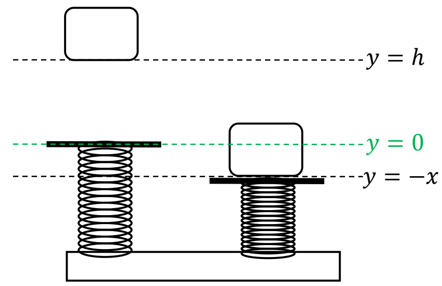
8.10

a) Only $\vec{v} = 0$ at max stretch or max compression.

b) If we consider the mass and spring as a system $W_{non-con} = 0$.

I set my reference level as initial height of the platform.

You are free to choose another option; the results should be the same.



$$\frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2$$

$$mgh = mg(-x) + \frac{1}{2}kx^2$$

$$k = \frac{2mg(h+x)}{x^2}$$

Note1: The normal force of the block on the platform does positive work on the platform while the normal force of the platform on the block does negative work on the block. There is no net work done on the mass spring system by the normal force between the platform and the block because it is an internal force.

In general, internal forces do no *net* work on a system.

Note2: typically we reference gravitational potential energy to *the center of mass* of the falling object. However, for objects moving in straight lines it makes no difference in the equations to shift our reference point to the bottom or top of the object.

Note3: if the platform and spring had non-negligible mass, we would need to do a momentum problem during the collision. Why? Energy is lost in most real-life collisions. In this special case, (massive block hits light platform/spring), a negligible amount of energy is lost. This is discussed thoroughly in the next chapter.

c) The same energy equation applies. Rearrange it to set up the quadratic formula to solve for x. Note: it is convenient to arrange the equation like

$$\frac{1}{2}x^2 - \frac{mg}{k}x - \frac{mgh}{k} = 0$$

$$x = \frac{-\left(-\frac{mg}{k}\right) \pm \sqrt{\left(-\frac{mg}{k}\right)^2 - 4\left(\frac{1}{2}\right)\left(-\frac{mgh}{k}\right)}}{2\left(\frac{1}{2}\right)}$$

$$x = \frac{mg}{k} \pm \sqrt{\left(\frac{mg}{k}\right)^2 + \frac{2mgh}{k}}$$

To more easily see which root to take, factor out $\frac{mg}{k}$ giving

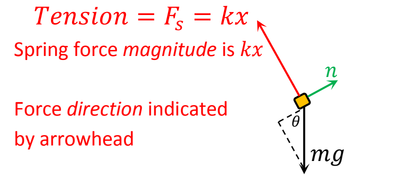
$$x = \frac{mg}{k}\left(1 \pm \sqrt{1 + \frac{2kh}{mg}}\right)$$

We see now we must use the positive root to make x positive. Remember, in our figure earlier we said $y_f = -x$ where x is the *distance* of max compression and made the minus sign explicit.

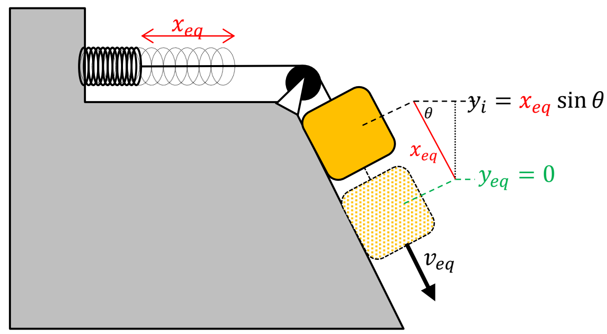
8.11

a) An FBD reveals $kx_{eq} = mg\sin\theta$ or $x_{eq} = \frac{mg}{k}\sin\theta$.

At the equilibrium position $a = 0$ but $v \neq 0$.



b) The system includes the mass, spring, and string.
Tension is an internal force and does no net work.
Normal force is perpendicular to displacement & does no work. The point connecting the spring to the wall undergoes no displacement so the force from the wall on the spring also does no work. We have a conservative system.



$$E_i = E_f$$

$$\frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2$$

$$mgx_{eq}\sin\theta = \frac{1}{2}mv_{eq}^2 + \frac{1}{2}kx_{eq}^2$$

$$v_{eq} = \sqrt{2gx_{eq}\sin\theta - \frac{k}{m}x_{eq}^2}$$

$$v_{eq} = \sqrt{2g\left(\frac{mg}{k}\sin\theta\right)\sin\theta - \frac{k}{m}\left(\frac{mg}{k}\sin\theta\right)^2}$$

$$v_{eq} = \sqrt{\frac{mg^2}{k}\sin^2\theta}$$

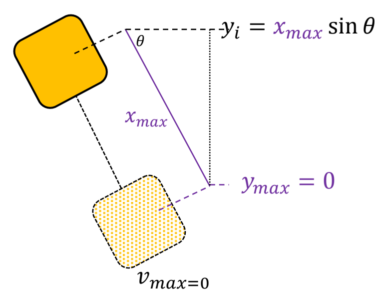
c) Speed is zero at max stretch. The others are non-zero.

d) The figure is similar to the one shown for part b with one exception:

at max stretch $v = 0$ but $a \neq 0$!

Think: just like when vertical projectile reaches max height $v = 0$ but $a \neq 0$. **Use energy (not forces).**

I chose to make the reference level @ max stretch for this part to clean up the math.



$$mgx_{max}\sin\theta = \frac{1}{2}kx_{max}^2$$

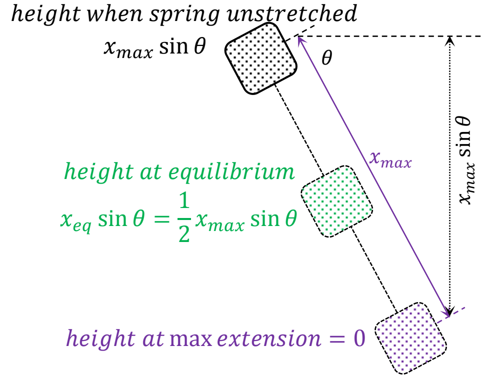
$$x_{max} = \frac{2mg}{k}\sin\theta$$

$$\boldsymbol{x_{max} = 2x_{eq}}$$

e) An FBD reveals $F_{net} = kx_{max} - mg\sin\theta = +mg\sin\theta$ (+ indicates directed up the plane).
At the instant the mass is released from rest the force is $-mg\sin\theta$ ($-$ indicates directed down the plane).
The problem is symmetric about the equilibrium position!

Solution for Part f is on the next page…

f) Total energy is constant since $W_{non-con} = 0$.
I choose to set my reference height as zero at max extension.



Initially the spring is unstretched and the mass is at rest.
Therefore max gravitational potential energy is also total energy!

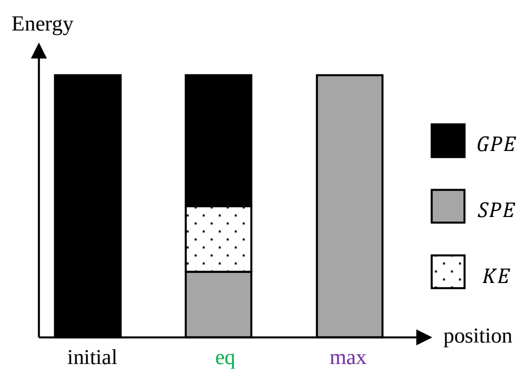
$$E_{total} = mgx_{max}\sin\theta = mg\left(\frac{2mg}{k}\right)\sin\theta = \frac{2m^2g^2}{k}\sin^2\theta$$

At max extension we have no gravitational potential energy and the object is instantaneously at rest (it is about to reverse direction).
Therefore max spring potential energy is also total energy.

$$E_{total} = \frac{1}{2}kx_{max}^2 = \frac{1}{2}k\left(\frac{2mg}{k}\sin\theta\right)^2 = \frac{2m^2g^2}{k}\sin^2\theta$$

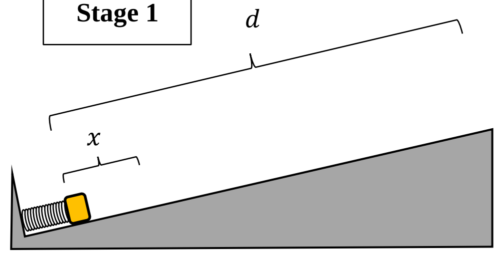
The hard part is figuring out the energy split at equilibrium. At equilibrium we know

Energy term	**Ratio with E_{tot}**
$GPE_{eq} = mgx_{eq}\sin\theta$	$\frac{GPE_{eq}}{E_{tot}} = \frac{mgx_{eq}\sin\theta}{mgx_{max}\sin\theta} = \frac{x_{eq}}{x_{max}} = \frac{1}{2}$
$SPE_{eq} = \frac{1}{2}kx_{eq}^2 = \frac{1}{2}\frac{m^2g^2}{k}\sin^2\theta$	$\frac{SPE_{eq}}{E_{tot}} = \frac{\frac{1}{2}\frac{m^2g^2}{k}\sin^2\theta}{\frac{2m^2g^2}{k}\sin^2\theta} = \frac{1}{4}$
$K_{eq} = \frac{1}{2}mv_{eq}^2 = \frac{1}{2}\frac{m^2g^2}{k}\sin^2\theta$	$\frac{K_{eq}}{E_{tot}} = \frac{\frac{1}{2}\frac{m^2g^2}{k}\sin^2\theta}{\frac{2m^2g^2}{k}\sin^2\theta} = \frac{1}{4}$

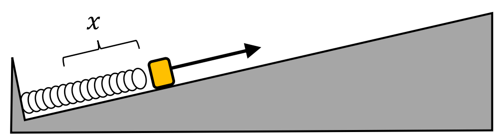


8.11½ A set of three stage pictures are shown at right.
I include stage 2 for reference, but it is not needed in the computation.

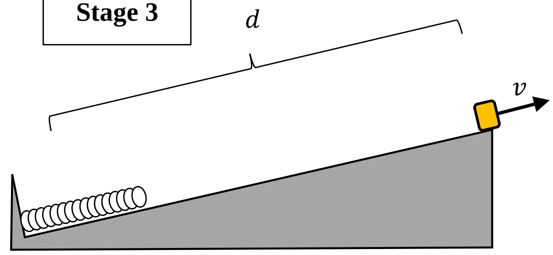
Stage 1



Stage 2



Stage 3



Stage 2 shows the instant the block separates from the spring.
It turns out this occurs just after the spring returns to its unstretched length!

Think: once the block goes past its unstretched position, the spring itself experiences a force to the down the plane (trying to restore the spring to its unstretched length).

It is at this moment (spring at unstretched length) the block *typically* separates from the spring. With significant kinetic friction present, the spring's momentum keeps the spring in contact with the block for a tiny amount of additional distance. Since we typically assume the spring has negligible mass, we can ignore any additional push up the hill from the spring if it doesn't separate at the exact instant it reaches the equilibrium position.

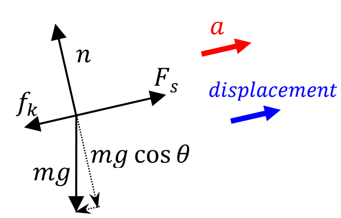
Rather than worry about Stage 2, directly compare **Stage 1** to **Stage 3**.

$$\frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2 + W_{ext/n.c.} = \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2$$

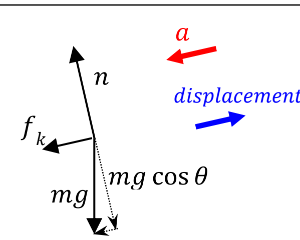
$$0 + 0 + \frac{1}{2}kx^2 + W_{friction} = \frac{1}{2}mv^2 + mgd\sin\theta + 0$$

To get the $W_{friction}$, consider the FBD's shown below Stage 3.

FBD while in contact with spring (and moving)



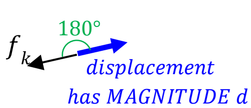
FBD after separation from spring



Notice, trying to solve this type of problem with forces is very complicated. As the block travels the initial distance x (while still in contact with the spring) the magnitude of spring force is <u>not</u> constant. This implies the acceleration is not constant either!
Furthermore, we see the direction of acceleration changes after the block separates from the spring!

HOWEVER, the work done by friction is actually fairly easy to compute!
In both FBD's one notices $n = mg\cos\theta$.
Since, the block is moving relative to the incline in each FBD the friction in each FBD is $f_k = \mu_k n = \mu_k mg\cos\theta$.
Since work and friction point opposite directions we find

$$W_{friction} = f_k d\cos 180° = -\mu_k mgd\cos\theta$$



Plugging in gives

$$\frac{1}{2}kx^2 + (-\mu_k mgd\cos\theta) = \frac{1}{2}mv^2 + mgd\sin\theta$$

$$\boldsymbol{k = \frac{1}{x^2}\{mv^2 + 2mgd(\sin\theta + \mu_k\cos\theta)\}}$$

8.12

a) See plots below. Units left off for brevity. Notice the cool double bump feature of the kinetic energy. It hits max speed *twice* during each oscillation (once on the way down and again on the way back up).

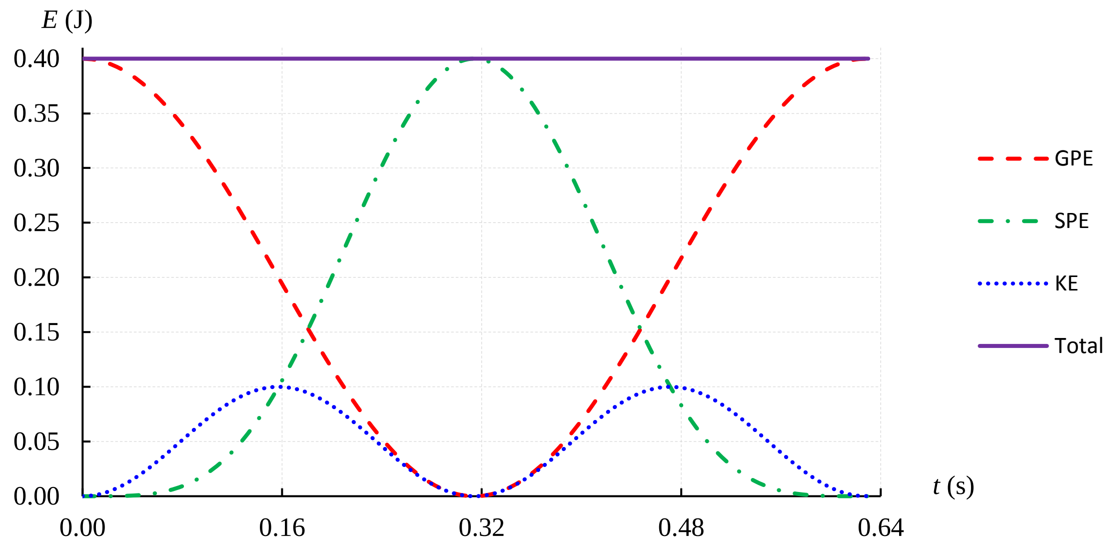
b) The deriviative of position gives *velocity* $v = -1\sin(10t)$. The *speed* is $v = 1|\sin(10t)|$. Fortunately, kinetic energy uses speed squared which allows us to ignore the distinction. At any instant the spring is streched an amount $x = 0.2 - y(t) = 0.1(1 - \cos(10t))$. Using the above information we find

$$U_G(t) = mgy = 0.2(1 + \cos(10t))$$

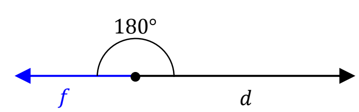
$$K(t) = \frac{1}{2}mv^2 = 0.1\sin^2(10t)$$

$$U_S(t) = \frac{1}{2}kx^2 = 0.1(1 - \cos(10t))^2 = 0.1(1 - 2\cos(10t) + \cos^2(10t))$$

$$E_{total} = U_G(t) + K(t) + U_S(t) = 0.4$$



8.13 Use $E_i + W_{non-con} = E_f$. Normal force does no work ($\perp$ to displacement). Friction does do work! From an FBD it is straightforward to find $n = mg$. Since the block is moving relative to the floor $f = \mu_k n = \mu_k mg$. Sketching a little picture of the friction force and displacement helps determine the angle between is 180°. To be clear, the distance traveled is d but the displacement vector is $\Delta\vec{x} = d\hat{\imath}$. The work done by friction is thus



$$W_{friction} = fd\cos(180°) = -\mu_k mgd$$

Now we do the energy problem

$$E_i + W_{non-con} = E_f$$

$$\frac{1}{2}mv_i^2 + mgh_i + W_{friction} = \frac{1}{2}mv_f^2 + mgh_f$$

In this problem the initial height and final height are the same. Any time this happens the mgh's drop. Also, final speed is zero as the block comes to rest.

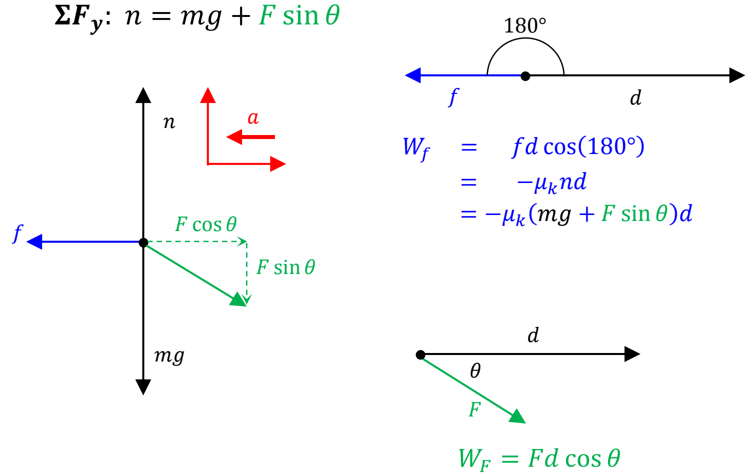
$$\frac{1}{2}mv^2 + (-\mu_k mgd) = 0$$

$$\mu_k = \frac{v^2}{2dg}$$

8.14 The FBD is shown at right. Also shown are calculations of the work done by friction and the applied force $\vec{F}$. Notice the applied force does positive work since it has a component pointing parallel to displacement. Friction again does negative work as it points opposite displacement.
If we ignore gravitational potential energy (because level ground) we find

$$E_i + W_{non-con} = E_f$$
$$\frac{1}{2}mv^2 + W_f + W_F = 0$$
$$\frac{1}{2}mv^2 + (-\mu_k(mg + F\sin\theta)d) + Fd\cos\theta = 0$$
$$F = m\frac{\frac{v^2}{2d} - \mu_k g}{\mu_k \sin\theta - \cos\theta}$$

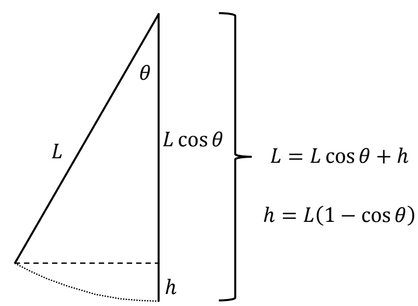
ΣF_y: $n = mg + F\sin\theta$



b) Damn those zombies….I ran out of brains.
8.15 Do the sim.

8.16
a) At any point in the motion, tension points perpendicular to displacement. In circular motion, a force towards the center can change the direction of velocity but not speed. Forces towards the center in circular motion problems do no work! The picture at right helps determine the initial height as $h = L(1-\cos\theta)$. This assumes the final height is zero.

$$mgh = \frac{1}{2}mv_f^2$$
$$v_f = \sqrt{2gh} = \sqrt{2gL(1-\cos\theta)}$$



b) An FBD at the bottom gives

$$T = mg + ma_c = T = mg + m\frac{v^2}{L} = mg(3 - 2\cos\theta)$$

Think: if $\theta = 0°$ the ball is just hanging there. If the ball just hangs there we should get $T = mg$. Check!

8.17 The result of the previous problem holds giving $v_{max} = \sqrt{2gL}$.
Doing the energy problem from the top $h_i = L$ to some arbitrary angle $h_f = L(1-\cos\theta)$ gives

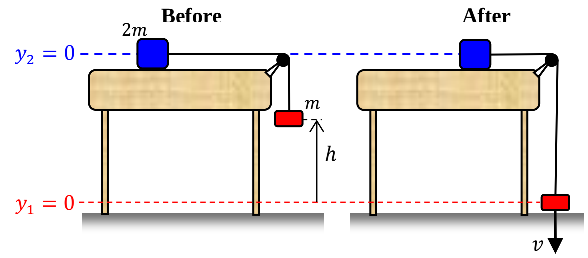
$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$$
$$mgL = \frac{1}{2}mv_f^2 + mgL(1-\cos\theta)$$
$$2gL = v_f^2 + 2gL(1-\cos\theta)$$
$$v_f^2 = 2gL\cos\theta$$

The problem statement tells us $v_f = \frac{v_{max}}{2} = \sqrt{\frac{gL}{2}}$. Plug in and solve for θ.

$$\theta = \cos^{-1}\left(\frac{1}{4}\right) \approx 76°$$

You get half of the speed in the first 14°!

8.18



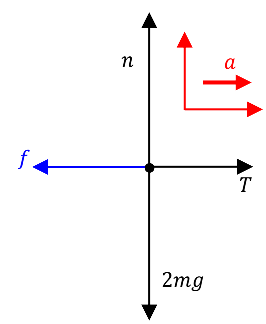
a) The general energy equation for a 2 mass system is

$$K_{1i} + K_{2i} + U_{1i} + U_{2i} + W = K_{1f} + K_{2f} + U_{1f} + U_{2f}$$

I assign separate reference levels for each block.

Any offset associated with one object being higher always cancels in the energy equation because the same offset appears in h_i and h_f. I explain why this is ok in another way in the next answer…

FBD $2m$



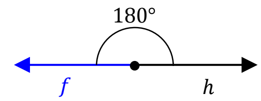
In this problem

- Tension is internal ⇒ no work
- Normal is perpendicular to displacement ⇒ no work
- Pulley forces on string do not displace string ⇒ no work
- $W_{friction} = \vec{f} \cdot \Delta\vec{x} = (\mu_k n)(h)\cos 180° = -2\mu_k mgh$ (see FBD at right)
- Both blocks have same final speed
- Mass $2m$ moves on level ground; can ignore GPE for $2m$

$$K_{1i} + K_{2i} + U_{1i} + U_{2i} + W = K_{1f} + K_{2f} + U_{1f} + U_{2f}$$

$$mgh - 2\mu_k mgh = \frac{1}{2}(m + 2m)v_f^2$$

$$h = 3.75\frac{v^2}{g}$$



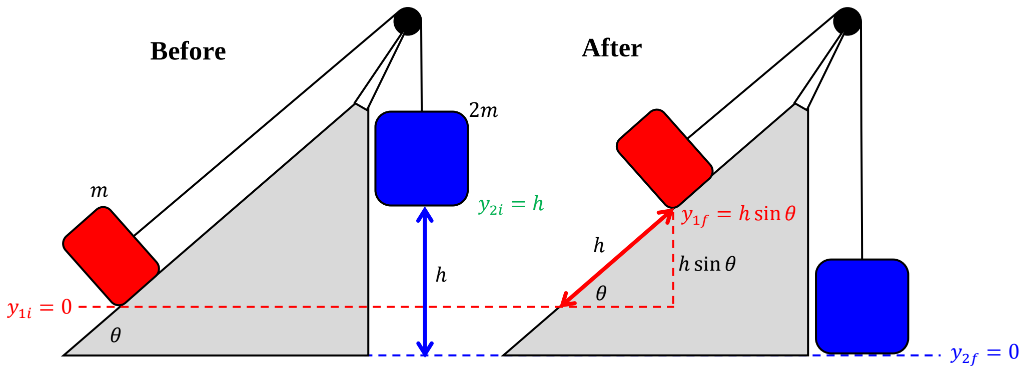
$$\begin{aligned} W_f &= fd\cos(180°) \\ &= -\mu_k nd \\ &= -\mu_k(2mg)d \end{aligned}$$

b) This is pretty tricky.
The wording indicates the problem makes no sense if μ_s gets too large.
Why?
If static friction coefficient gets too large, it will prevent the blocks from sliding!
This implies you should do a force problem (setting $a = 0$ & $f = \mu_s n$ for block $2m$).
From FBDs and the force equations I found the blocks remain at rest when $\mu_s \geq 0.5$.

8.19

a) I think a big picture helps on this one. Note: for blocks along an incline the change in height usually ends up being $L\sin\theta$. In this case the blocks moved distance h so we know $h = L$.

When you have two blocks like this, give each one its own reference level. This is totally legit because doing this gives the correct *change* in potential energy for each block. In the end, only changes in potential energy are physically meaningful. The choice of reference level ($y = 0$) is arbitrary and thus can be different for each block.



Problem said friction was negligible. That implies $\mu_k \approx 0$ and thus $W_{friction} = 0$.
Normal force is perpendicular to displacement. That implies $W_{normal} = 0$.
Tension is an internal force. It does positive work on m but the same amount of negative work on $2m$.
Together these facts tell us $\underset{n.c.}{W_{ext}} = 0$.
Since both blocks start from rest we know $K_{1i} = K_{2i} = 0$.
Because the blocks are tied together with a string we assume they share the same speed $v_{1f} = v_{2f} = v_f$.

$$K_{1i} + K_{2i} + U_{1i} + U_{2i} + W = K_{1f} + K_{2f} + U_{1f} + U_{2f}$$

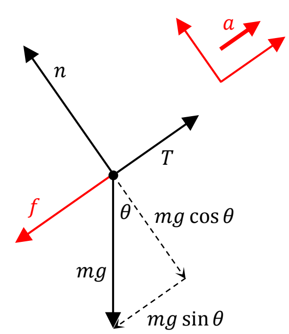
$$2mgh = \frac{1}{2}(m + 2m)v_f^2 + mgh\sin\theta$$

$$\frac{3}{2}mv_f^2 = 2mgh - mgh\sin\theta$$

$$\frac{3}{2}v_f^2 = gh(2 - \sin\theta)$$

$$v_f = \sqrt{\frac{2gh}{3}(2 - \sin\theta)}$$

FBD m



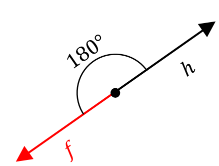
b) If friction is non-negligible, add work done by friction using the FBD shown at right.

$$2mgh - \mu_k mgh\cos\theta = \frac{1}{2}(m + 2m)v_f^2 + mgh\sin\theta$$

$$\frac{3}{2}mv_f^2 = 2mgh - mgh\sin\theta - \mu_k mgh\cos\theta$$

$$\frac{3}{2}v_f^2 = gh(2 - \sin\theta - \mu_k\cos\theta)$$

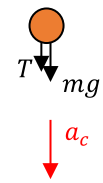
$$v_f = \sqrt{\frac{2gh}{3}(2 - \sin\theta - \mu_k\cos\theta)}$$



$$\begin{aligned} W_f &= fd\cos(180°) \\ &= -\mu_k nd \\ &= -\mu_k(mg\cos\theta)d \end{aligned}$$

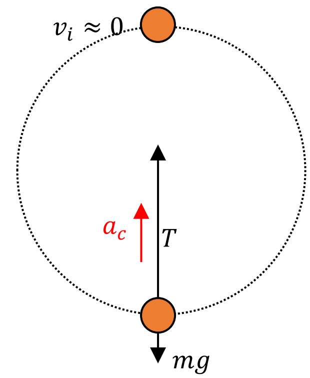
8.20

a) For a ball on a string in a vertical circle, the string exerts a tension force towards the circle's center. Because tension cannot be negative on the string, one can set $T = 0$ at the top.
Said another way, tension in the string cannot push outwards to keep the ball in circular motion.
Doing an FBD gives $v_{min} = \sqrt{rg}$.
When the ball is attached to a stiff rod, the rod *can* exert an outwards force to keep the ball in circular motion. In this case $v_{min} = 0$.



b) If we set the *bottom* of the vertical loop as our GPE reference level, then $h_i = 2r$ & $h_f = 0$.

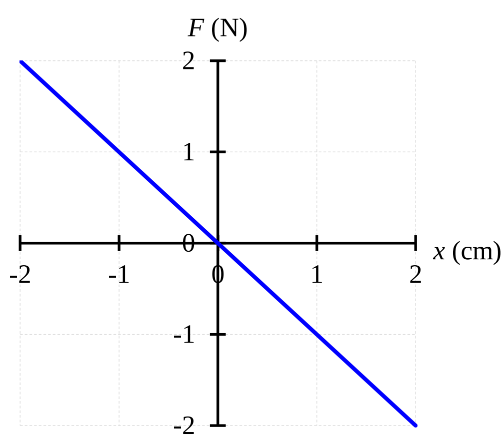
$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$$
$$v_f = \sqrt{4rg}$$
$$T_{bottom} - mg = ma_c$$
$$T_{bottom} - mg = m\frac{\left(\sqrt{4rg}\right)^2}{r} = 4mg$$
$$\boldsymbol{T_{bottom} = 5mg}$$

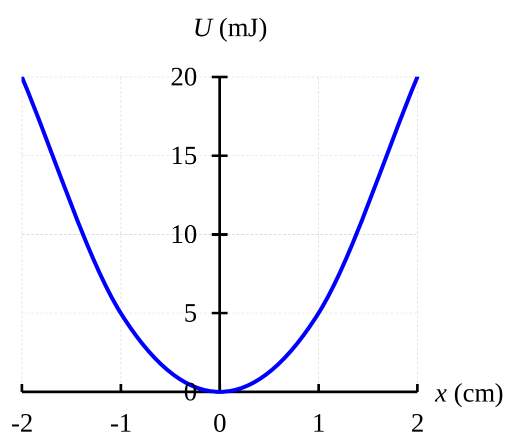


8.21 The plots are shown below.
Dealing with unit prefixes in plots is <u>definitely</u> good practice.
Ideally, prefixes are chosen to minimize the number of leading zeroes on each axis to reduce clutter.

Using scientific notation for everything is easier for whomever *creates* the graph.
Using scientific notation is exhausting for whoever *reads* the graph.
It is polite to make things easier for people who will read your work.

Side note: most engineers would probably frown upon my usage of centimeters.
While a convenient unit for many experiments due to its size, engineers would likely use millimeters instead.





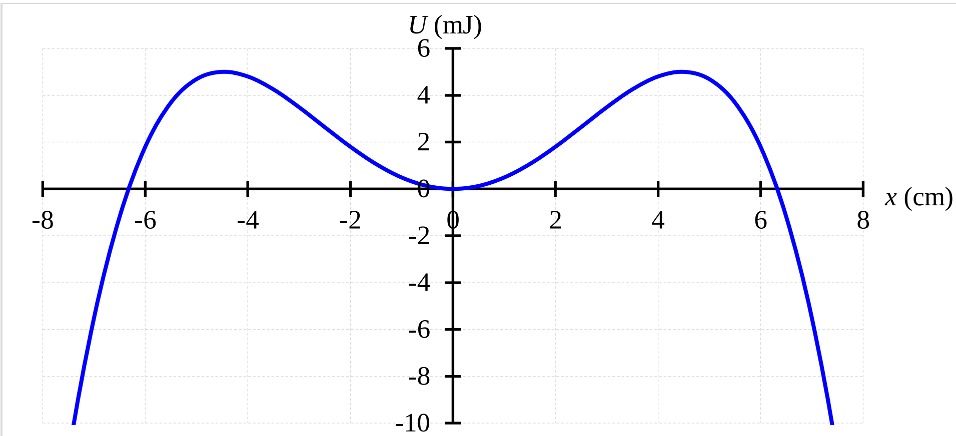
8.22

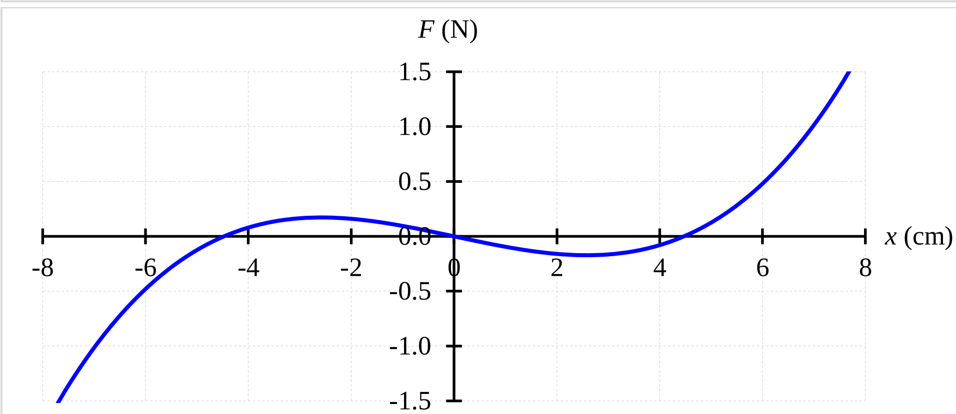
a) $U(x) = \frac{1}{2}kx^2 - \frac{1}{4}bx^4$

b) $x = 0, \pm\sqrt{\frac{2k}{b}}$

c) $x = 0, \pm\sqrt{\frac{k}{b}}$…remember that equilibrium positions occur when $F = 0$, not $U = 0$.

d) Note: these forces are way too large to be realistic (for atomic problems)…See below





8.23

a) Answer in problem statement.
b) Tension in the zombie rope does $+$ then $-$ work (but not the same amount in each direction).
c) Gravity does $-$ then $+$ work.
d) Friction does $-$ work in both directions! Normal force does no work in either direction.
e) We could use potential energy instead of computing the work done by gravity (but not both).
Notice we *cannot* define potential energy for the tension force as it is not, in general conservative. It does different amounts of work going in each direction. The *net* work for a closed path (same starting and ending point) is *not* zero (in general) for external tension forces. This indicates the work done by tension is NON-conservative.

8.24 I was lazy with sig figs in this solution to focus on content.

a) $[\sigma] = \mathrm{m}$ and $[\varepsilon] = \mathrm{J}$. It helps to look first at the term inside the parentheses. Anytime you add things together they must have the same units. If for some reason the first term had units, the second term would then have the square root of those units. The only way these two terms could have the same units is if both terms are unitless. Once you see this the problem goes…
b) $F(x) = 24\varepsilon\left(2\frac{\sigma^{12}}{x^{13}} - \frac{\sigma^6}{x^7}\right)$
c) To find equilibrium positions, set $F(x) = 0$ and solve for x. I found $x_{eq} = 2^{1/6}\sigma \approx 1.12\sigma$
d) The upper plot is U vs x. Notice in the lower plot $F_x = 0$ at $x_{eq} \approx 1.12\sigma$.
e) The force is *large* and *positive* (to the right) for $x < x_{eq}$.
The force is *large* and *negative* (to the left) for $x_{eq} < x \lesssim 1.2\sigma$
The force is negative for all $x > x_{eq}$.
Stable equilibrium at $x_{eq} \approx 1.12\sigma$. For large x the system is approximately in neutral equilibrium.

Try a web search for a simulation of atomic interactions to understand what the parameters σ & ε represent.

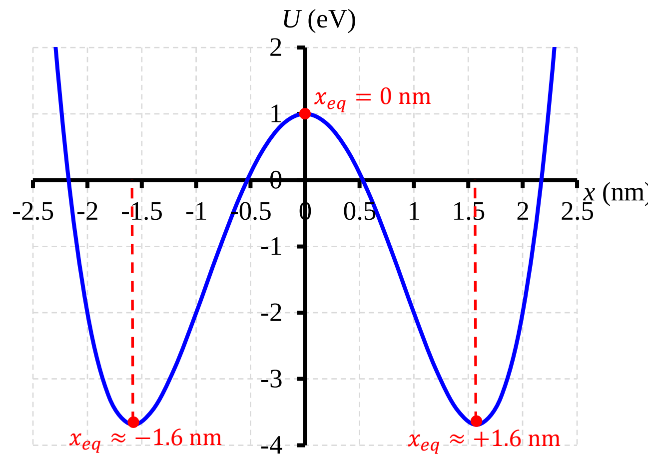
8.25 I was lazy with sig figs & units in this solution to focus on content.

a) It would roll to the right; force to the right.
b) It would roll to the left; force is negative at $x = 2$…makes sense.
c) The slope is steeper at $x = 1$ than at $x = 2$. The force (and acceleration) are greater at $x = 1$.

8.26

a) Equilibrium points occur wherever the slope of $U(x)$ is zero. The answers can be read directly off the graph to two sig figs. Alternatively, I used part b to get more sig figs (see below).

The equilibrium point at $x = 0$ is *unstable*. Any small displacement from this equilibrium position and the particle (I'm assuming an electron) moves farther from equilibrium rather than trying to return to it. This makes sense: the electron is halfway between the two atoms. If it moves closer to one nuclei, that nucleus wins the tug of war for the electron. Using the marble analogy, a marble at the top of a hill will always roll away from the summit (unstable equilibrium).



At $x \approx \pm 1.6$ nm we see *stable* equilibrium points. Any small displacement from these equilibrium positions and the particle returns to the equilibrium position. A marble at the bottom of a bowl will roll back and forth and oscillate about the stable equilibrium.

b) I found $F(x) = -3.00x^3 + 7.50x$.
Side note: by setting $F(x) = 0$ and solving for x one finds more precise values for the equilibrium positions.

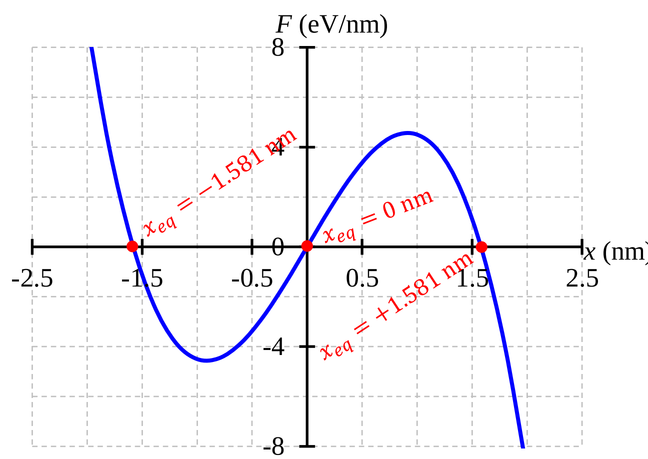
$$-3.00x^3 + 7.50x = 0$$
$$x(-3.00x^2 + 7.50) = 0$$

We see $x_{eq} = 0$ is one solution.
We also see $-3.00x^2 + 7.50 = 0$ gives two other solutions at $x_{eq} = \pm 1.581$ nm.

c) See plot at right and read numbers off the plot.

d) The units for this force equation are $\frac{\text{eV}}{\text{nm}}$. Note:

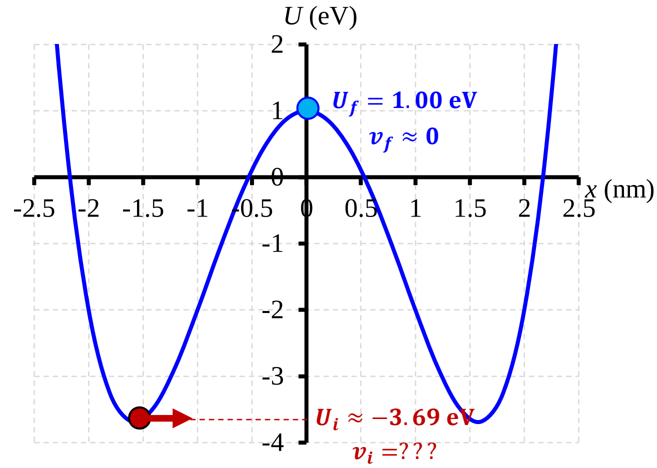
$$1\frac{\text{eV}}{\text{nm}} \times \frac{1\text{ nm}}{10^{-9}\text{m}} \times \frac{1.6 \times 10^{-19}\text{J}}{1\text{ eV}} \times \frac{1\text{ N}\cdot\text{m}}{1\text{ J}}$$
$$= 1.6 \times 10^{-10}\text{N} \approx 160\text{ pN}$$



e) Plot of *F* vs *x* is shown at right. Remember, in this plot force uses $\frac{\text{eV}}{\text{nm}}$ and position uses nm.

Part f on next page…

f) First consider the potential energy graph shown at right.



The particle is initially at $x_{eq} = -1.581$ nm where

$$U_i = 0.750x^4 - 3.75x^2 + 1.000$$
$$U_i = 0.750(-1.581)^4 - 3.75(-1.581)^2 + 1.000$$
$$U_i \approx -3.69 \text{ eV} = -5.91 \times 10^{-19} \text{ J}$$

Think about the ball analogy now.

To get from the left well into the right well, the ball must have enough initial kinetic energy to roll over the hill at $x = 0$ where $U_f = +1.00 \text{ eV} = 1.602 \times 10^{-19}$ J.

To get over the hill, the ball must still be moving at the top of the hill, but even the tiniest amount of speed at $x = 0$ will allow it to continue on to the bottom of the next well.

If we assume there are no external or non-conservative forces acting on the electron we may write conservation of energy.

$$K_i + U_i = K_f + U_f$$
$$K_i - K_f = U_f - U_i$$
$$K_i - K_f = \Delta U$$
$$\frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2 = \Delta U$$
$$v_i = \sqrt{\frac{2}{m}\Delta U}$$

Since I solved for initial *speed* I used the positive root.

I will use $\Delta U = +4.69 \text{ eV} = 7.51 \times 10^{-19}$ J.

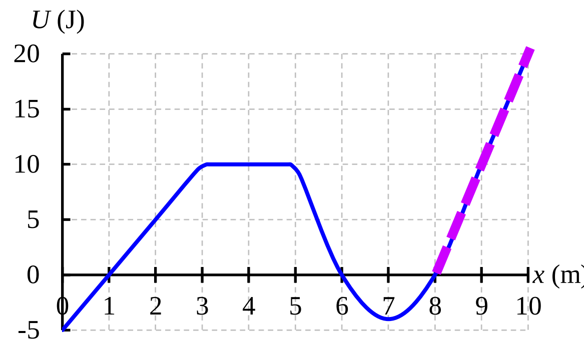
My electron mass is a number I can look up in a table (on a test this number would be provided).

I found $m_e = 9.11 \times 10^{-31}$ kg.

$$v_i = \sqrt{\frac{2}{9.11 \times 10^{-31} \text{ kg}}(7.51 \times 10^{-19} \text{ J})}$$
$$v_i = \sqrt{\frac{2}{9.11 \times 10^{-31} \text{ kg}}(7.51 \times 10^{-19} \text{ N} \cdot \text{m})}$$
$$v_i = \sqrt{\frac{2}{9.11 \times 10^{-31} \text{ kg}}\left(7.51 \times 10^{-19} \text{ kg} \cdot \frac{\text{m}}{\text{s}^2} \cdot \text{m}\right)}$$
$$\boldsymbol{v_i \approx 1.284 \times 10^6 \frac{m}{s}}$$

Notice it is important to use appropriate units when you plug in numbers!!!

8.27 **I was lazy with sig figs in this solution to focus on content.**



a) Use

$$F_x = -slope = -\frac{rise}{run} \approx -10\text{ N}$$

There is often confusion between $\vec{F}$, F_x, and magnitude F.
Recall, for a 1D force $\vec{F} = F_x\hat{\imath}$.
Consider writing $\vec{F} = (-10\text{ N})\hat{\imath}$ to be clear.

b) Force is directed to the right for $x = 5$ to 7 m.
Notice a *negative* slope gives a force to the *right*.

c) Force is directed to the left for $x = 0$ to 3 m and $x = 7$ to 10 m.
Notice a *positive* slope gives a force to the *left*.

d) *Neutral* equilibrium for $x = 3$ to 5 m; *stable* equilibrium at $x = 7$ m.
Note: one could probably argue the equilibrium from 3 to 5 m is better classified as unstable since a small perturbation would eventually cause the particle to move away and never come back.
My thought process was we have an extended region of space with zero slope (as opposed to a single point in space with zero slope).

Typically I call the extended regions with zero slope neutral equilibrium even when the particle may eventually never come back. Why? A small perturbation of the position in that region of space (except at the region's endpoints) still leaves you in equilibrium. That is the essence of neutral equilibrium.

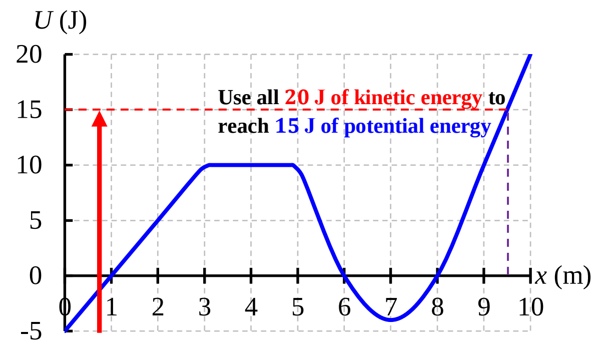
Being realistic for a moment, most interesting equilibrium positions end up being either stable or unstable.

8.28

a) At the origin ($x_i = 0$) initial **potential** energy is $U_i = -5\text{ J}$.
Because we have a conservative force

$$E_{total} = 15\text{ J} = K + U = constant$$
$$E_{total} = 15\text{ J} = K_i + (-5\text{ J})$$

From this one finds initial **kinetic** energy is $K_i = 20\text{ J}$.



Since the particle has 20 J of kinetic energy, it has enough kinetic energy to pass the trouble spot between $x = 3 \rightarrow 5$ m.

Then the particle will keep going as moving to the right as long as kinetic energy is larger than zero.
Notice what happens when the particle reaches position $x = 9.5$ m.
At this instant $K_f = 0$ and $U_f = E_{total} = 15$ J.
Therefore the particle reaches $x = 9.5$ m before reversing direction & travelling past the origin moving left.

b) Use

$$K_i + U_i = E_i$$
$$K_i = E_i - U_i$$
$$\frac{1}{2}mv_i^2 = E_i - U_i$$
$$v_i = \sqrt{\frac{2}{m}(E_i - U_i)}$$
$$v_i = \sqrt{\frac{2}{0.40\underline{0}\text{ kg}}\left(1\underline{5}\text{ J} - (-5.\underline{0}\text{ J})\right)} = 1\underline{0}\frac{\text{m}}{\text{s}}$$

c) At $x = 5.5$ m one sees $U_f = +5.0$ J. Going from the origin to $x = 5.5$ m gives $\Delta U = +10.0$ J.

$$K_i + U_i = K_f + U_f$$
$$K_f = K_i - \Delta U$$
$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 - \Delta U$$
$$v_f = \sqrt{v_i^2 - \frac{2\Delta U}{m}}$$
$$v_f = \sqrt{(1\underline{0})^2 - \frac{2(10.\underline{0})}{0.40\underline{0}}} = \sqrt{5\underline{0}} = 7.\underline{0}7 = 7.1\frac{\text{m}}{\text{s}}$$

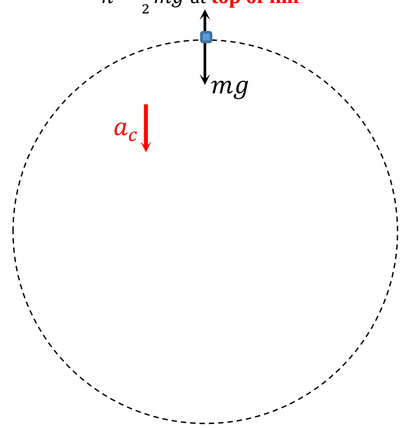
d) If released from rest total energy equals initial potential energy. The object initially moves left (force is opposite the slope of U vs x). It will not have enough energy to reach $x = 5$ m. It comes to rest for a split second and then moves back to the initial position. The mass oscillates between $x = 8.9$ m and $x = 5.1$ m.

e) Again the mass will start out moving left. This time the mass has enough energy to reach $x = 5$ m with speed close to zero. The mass slowly creeps leftwards towards $x = 3$ m. Once over the potential barrier, the mass picks up speed as it races towards the origin.

Note: The region between $x = 3$ to 5 m is sometimes called a "potential barrier". Eventually you may learn about quantum tunneling. In quantum tunneling, the electric field surrounding a molecule creates electrical potential barrier. Electrons with total energy slightly less than the barrier can escape with a probability that decreases in proportion to the difference between the potential barrier and the total energy. One practical application is the scanning tunneling microscope. This device can be used to draw pictures using single atoms as pixels or to study tiny biological materials such as DNA. Check it out with a web search.

8.29

Problem statement tells us $n = \frac{1}{2}mg$ at **top of hill**



a) From an FBD **at the top** of hill (see figure at right) one finds

$$mg - n = ma_c$$

$$mg - \left(\frac{1}{2}mg\right) = m\frac{v^2}{r}$$

$$\frac{1}{2}mg = \frac{mv^2}{r}$$

$$\frac{1}{2}g = \frac{v^2}{r}$$

$$\boldsymbol{v_{top\ of\ hill} = \sqrt{\frac{Rg}{2}}}$$

Next do an energy problem comparing initial position to the top of the hill.

$$\frac{1}{2}mv_i^2 + mgh + W_{n.c/ext} = \frac{1}{2}mv_{top\ of\ hill}^2 + mgh_{top\ of\ hill}$$

$$0 + mgh + 0 = \frac{1}{2}m\left(\sqrt{\frac{Rg}{2}}\right)^2 + mg(2R)$$

From there I found $\boldsymbol{h = 2.25R}$.

b) I chose to do an energy problem comparing initial position to the bottom

$$\frac{1}{2}mv_i^2 + mgh + W_{n.c/ext} = \frac{1}{2}mv_{bot\ of\ hill}^2 + mgh_{bot\ of\ hill}$$

$$0 + mgh + 0 = \frac{1}{2}mv_{bot\ of\ hill}^2 + mg(0)$$

$$v_{bot\ of\ hill} = \sqrt{\frac{9Rg}{2}}$$

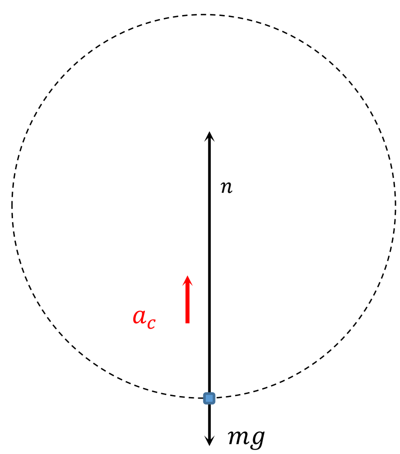
At the bottom the FBD (figure at right) gives $n - mg = ma_c$.

Notice this is not the same as the red equation above!!!

THINK: **at the top** of the hill the normal force points *away* from the center of the circle.

At the bottom the normal force points *towards* the center of the circle!!!

Plug in the speed into a_c and solve for n. I found $n = 5.5mg$ or 5.5 times the weight.



Solution continues on the next page…

c) When going over the top of a hill, riders losing contact is equivalent to $n = 0$.
The same FBD from part a applies but we can now set $n = 0$.

$$mg - n = ma_c$$

$$g = a_c$$

$$v_{lose\ contact} = \sqrt{Rg}$$

Now do an energy problem comparing the initial position to the top of the hill (solving for h).

$$\frac{1}{2}mv_i^2 + mgh + W_{n.c/ext} = \frac{1}{2}mv_{top\ of\ hill}^2 + mgh_{top\ of\ hill}$$

$$0 + mgh + 0 = \frac{1}{2}m\left(\sqrt{Rg}\right)^2 + mg(2R)$$

$$\boldsymbol{h = 2.5R}$$

Notice: this is an <u>increase of $0.25R$</u> from the result of part a…

A note on wording: we are asked "by what factor can height be increased?"

$$factor \times h_{before\ change} = h_{after\ change}$$

$$factor = \frac{h_{after\ change}}{h_{before\ change}}$$

$$factor = \frac{2.5R}{2.25R}$$

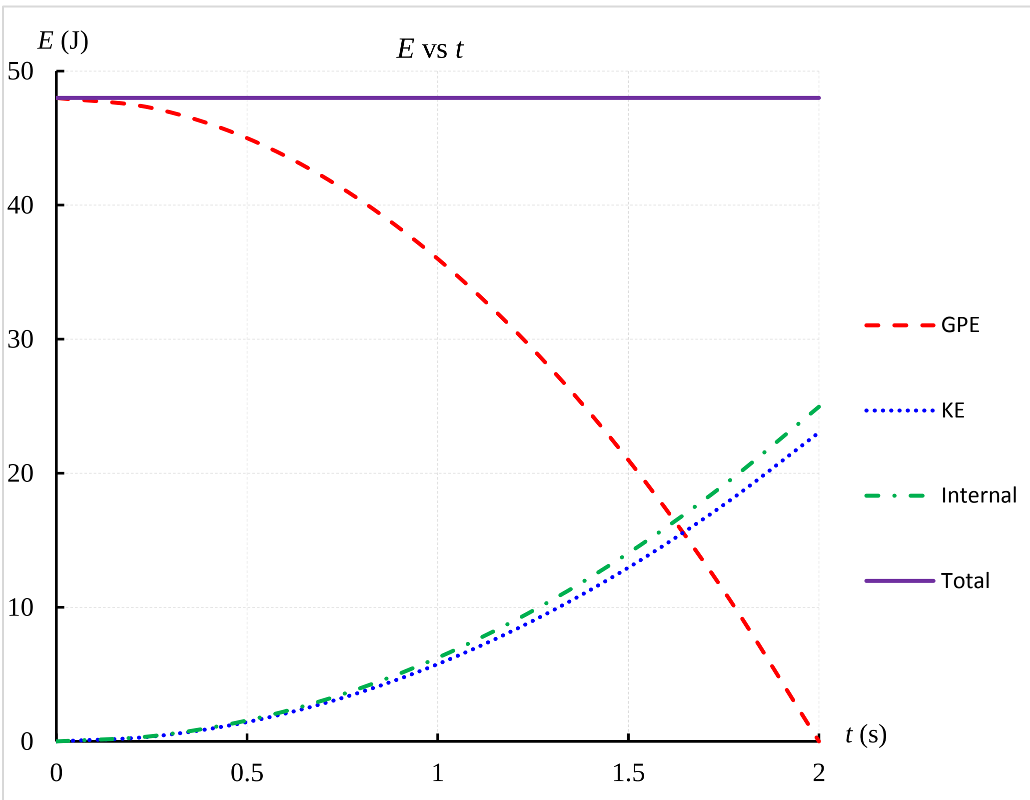
$$\boldsymbol{factor = 1.111}$$

8.30 **I left off units to reduce clutter in the solution…you would be expected to include units on final answers.**

a) See plots below.

b) From an FBD we find $a = g(\sin\theta - \mu_k \cos\theta) = 0.24g \approx 2.4$ (units left off for brevity).
The speed is thus $v = 2.4t$.
Assuming the final position is zero, the *position* on the track is given by $x = L - \frac{1}{2}at^2 = 4.8 - 1.2t^2$.
The *distance traveled* is $d = \frac{1}{2}at^2 = 1.2t^2$.
The *height* is $x\sin\theta = 2.4 - 0.6t^2$.

$$U_G(t) = mgy = 48 - 12t^2$$
$$K(t) = \frac{1}{2}mv^2 = 5.76t^2$$
$$\Delta E_{int} = |W_f| = \mu_k mgd\cos\theta = 6.24t^2$$
$$E_{total} = U_G(t) + K(t) + \Delta E_{int} = 48 \text{ J}$$



8.31 Lazy with sig figs once again. In general, include 3 sig figs unless otherwise specified.

a) At $x = -4, 0,$ and 4 m.

b) When it reaches $x = -2.0$ m, the net work on the mass is 10 J − 2 J = 8 J. Recall the net work is area under a force versus position graph. Since $W_{net} = \Delta K = K_f - K_i$ you can plug in numbers and find v.

$$W_{net} = \Delta K$$
$$W_{net} = K_f - K_i$$
$$W_{net} = K_f \text{ if initially at rest}$$
$$W_{net} = \frac{1}{2}mv_f^2$$
$$v_f = \sqrt{\frac{2W_{net}}{m}}$$

c) Starting form rest at $x = -6$ m. The force is initially positive. Work done on particle is area under curve. CHANGE in potential energy is negative area under curve. If it starts at rest, it will once again be at rest when net work is zero (or when change in potential energy is zero). Notice, as the block travels to the right it initially *gains* energy from work equal to 10 J (area under curve between -6 and -4). Then it *loses* 4 J travelling from -4 to 0 (negative area under curve). At this point the block has *gained* energy net and is therefore still moving to the right. From 0 to 2 the block *gains* an additional 6 J from work. Then from 2 to 4 it *gains* 12 J from work. Finally, from 4 to 6 it *loses* 12 J. The net energy is always positive. The block will keep moving to the right the entire time. Any time the work is positive the object is speeding up, and time the work is negative the object is slowing down.

d) We know the origin has no potential energy from the problem statement. In general we know

$$U_f = U_i - \int_i^f \vec{F} \cdot d\vec{s}$$

For a force only in the x-direction we know

$$U_f = U_i - \int_i^f F_x \, dx$$
$$U_f = U_i - (Area\ under\ Force\ vs\ position\ plot)$$

If we say $x_f = 0$ m then $U_f = 0$. Now set $x_i = -6$ m and we can find the potential energy at that point.

$$U_f = U_i - (Area\ under\ Force\ vs\ position\ plot)$$
$$0 = U_i - (6\ \text{J})$$
$$U_i = 6\ \text{J}$$

e) You can repeat the above procedure (part d) for a bunch of different positions (say each 1 m). Then you can tabulate the values of U and sketch the plot. This is similar to the process described in **2.17**.

8.32

a) Initially all energy is stored in gravitational potential energy.
Work is done by friction on the horizontal portion of the track.
Final energy, is all sp[ring potential energy.
Don't forget velocity is zero at max spring compression!
The energy equation looks like this

$$mgR - \mu_k mgx = \frac{1}{2}kx^2$$

Note: I'm assuming the size of the block is negligible compared to the radius of curvature of the track.

$$x_{max} = \frac{\mu_k mg}{k}\left(\sqrt{1 + \frac{2kR}{mg}} - 1\right)$$

If you have trouble with the quadratic formula, consult **8.10c**.

b) Verify answer information is true by doing an energy problem and FBD.

The block will remain at rest whenever the max compression distance is *less* than $\sqrt{\frac{2mgR}{k}}$.

8.33

a) We can ignore the spring portion as no energy is lost during the bounce.
While the block is sliding the normal force is $mg\cos\theta$ and the frictional force is $\mu_k mg\cos\theta$.
The energy equation is

$$mgL_0\sin\theta + W_f = mgL_1\sin\theta$$

While sliding (up or down) normal force is $mg\cos\theta$ and frictional force is $\mu_k mg\cos\theta$.
In both directions the work done by friction is negative giving

$$W_f = -\mu_k mg(L_0 + L_1)\cos\theta$$

Plugging into the energy equations gives

$$mgL_0\sin\theta - \mu_k mg(L_0 + L_1)\cos\theta = mgL_1\sin\theta$$
$$L_0\sin\theta - \mu_k(L_0 + L_1)\cos\theta = L_1\sin\theta$$
$$L_0\sin\theta - \mu_k L_0\cos\theta = L_1\sin\theta + \mu_k L_1\cos\theta$$

Divide all terms by $\cos\theta$

$$L_0(\tan\theta - \mu_k) = L_1(\tan\theta + \mu_k)$$
$$L_1 = L_0\frac{\tan\theta - \mu_k}{\tan\theta + \mu_k}$$

From there I found

$$\boldsymbol{L_0 - L_1 = L_0\frac{2\mu_k}{\tan\theta + \mu_k}}$$

b) The ratio is

$$\boldsymbol{\frac{L_1}{L_0} = \frac{\tan\theta - \mu_k}{\tan\theta + \mu_k}}$$

c) The ratio will be the same.
Essentially it loses the same % of max height after each bounce.

d) If the ramp has angle less than the critical angle (if $\mu_s > \tan\theta$) the block stops at max height.
Large angles and/or low friction situations should have the block and up touching the spring.

8.34

a) To solve this, do an FBD and assume the block is on the verge of slipping.

$$f - F_s = ma_x$$
$$\mu_s n - kx_0 = 0$$
$$x_0 = \boldsymbol{\mu_s}\frac{mg}{k}$$
$$x_0 = 0.6\frac{mg}{k}$$

b) Now we are told $x_0 = 2.5\frac{mg}{k}$.

$$U_i + K_i + W_{\substack{ext \\ non-con}} = U_f + K_f$$

Here potential energy is spring potential energy.

We need not worry about gravitational potential energy because the height of the block remains constant.

$$\frac{1}{2}kx_0^2 + 0 - \boldsymbol{\mu_k} mgx_0 = 0 + \frac{1}{2}mv_1^2$$
$$v_1 = \sqrt{\frac{kx_0^2}{m} - 2\mu_k g x_0}$$

Since $\boldsymbol{\mu_k = 0.5}$:

$$\boldsymbol{v_1 = \sqrt{\frac{kx_0^2}{m} - gx_0}}$$

At this point you can easily plug in $x_0 = 2.5\frac{mg}{k}$ and clean it up a bit.

Solution continues on the next page…

c) Using energy one finds

$$U_i + K_i + W_{\substack{ext \\ non-con}} = U_f + K_f$$

$$\frac{1}{2}kx_0^2 + 0 - \mu_k mg(x_0 + x_1) = \frac{1}{2}kx_1^2 + 0$$

Multiply all terms by $\frac{2}{k}$. Group all terms on the right side.

$$0 = x_1^2 + \frac{2\mu_k mg}{k}(x_0 + x_1) - x_0^2$$

$$0 = x_1^2 + \frac{2\mu_k mg}{k}x_1 + \frac{2\mu_k mg}{k}x_0 - x_0^2$$

Here I did the quadratic formula using $a = 1$, $b = \frac{2\mu_k mg}{k}$, & $c = \frac{2\mu_k mg}{k} - x_0^2$.

$$x_1 = \frac{-\frac{2\mu_k mg}{k} \pm \sqrt{\left(\frac{2\mu_k mg}{k}\right)^2 - 4(1)\left(\frac{2\mu_k mg}{k}x_0 - x_0^2\right)}}{2(1)}$$

$$x_1 = -\frac{\mu_k mg}{k} \pm \sqrt{\left(\frac{\mu_k mg}{k}\right)^2 - \left(\frac{2\mu_k mg}{k}x_0 - x_0^2\right)}$$

$$x_1 = -\frac{\mu_k mg}{k} \pm \sqrt{\left(\frac{\mu_k mg}{k}\right)^2 - \left(\frac{2\mu_k mg}{k}\right)x_0 + x_0^2}$$

$$x_1 = -\frac{\mu_k mg}{k} \pm \sqrt{\left(x_0 - \frac{\mu_k mg}{k}\right)^2}$$

$$x_1 = -\frac{\mu_k mg}{k} \pm \left(x_0 - \frac{\mu_k mg}{k}\right)$$

Note: in this equation x_1 is a supposed to be the distance of the final stretch (not final block position).
As such, the result should be positive.
This implies we must use the positive root.

$$x_1 = -\frac{\mu_k mg}{k} + \left(x_0 - \frac{\mu_k mg}{k}\right)$$

$$\boldsymbol{x_1 = x_0 - 2\frac{\mu_k mg}{k}}$$

Since $\boldsymbol{\mu_k = 0.5}$:

$$\boldsymbol{x_1 = x_0 - \frac{mg}{k}}$$

Had I been clever, I might have tried completing the square for each spring energy term…I wasn't.

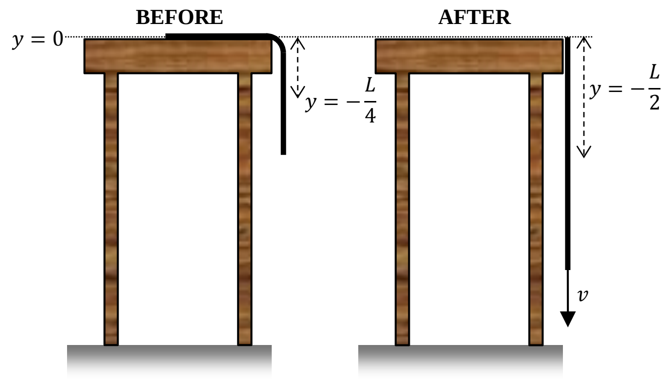
d) This also surprised me. In going from the *leftmost* position to the next *rightmost* position the energy equation gives

$$\frac{1}{2}kx_1^2 - \mu_k mg(x_1 + x_2) = \frac{1}{2}kx_2^2$$

The work is nearly identical to the previous part.
where x_2 is the max extension of the spring when the mass is at rest on the *right* side <u>for the 2nd time</u>.
The distance is decreased by an identical *amount* (not an identical *factor*!) as in the transit from right to left *even though less frictional work is done*!
In each transit from max extension to max compression (or vice versa) the maximum distance decreases by the amount $2\frac{\mu_k mg}{k} = \frac{mg}{k}$.
When the block returns to the right side it is therefore distance $x_2 = 0.5\frac{mg}{k}$.
It is at rest for an instant at this positon.
Notice this amount of extension does not provide enough spring force to overcome the maximum possible static friction.

8.35

a) I chose to let the table be my reference level.
Splitting up the rope into two equal parts: one on the table and one hanging.



Initially $\frac{m}{2}$ is centered at $y = -\frac{L}{4}$ while other half of the mass is supported so it does not contribute to the initial gravitational potential energy.

In the final state, all mass is centered at $y = -\frac{L}{2}$.
Conservation of energy yields

$$\frac{m}{2}g\left(-\frac{L}{4}\right) = \frac{1}{2}mv^2 + mg\left(-\frac{L}{2}\right)$$

$$v = \sqrt{\frac{3}{4}gL}$$

b) For a mass released from rest that falls distance $\frac{L}{2}$ we have $v = \sqrt{2g\left(\frac{L}{2}\right)} = \sqrt{gL}$.
It makes sense that this result is larger than our part a result.
Think: the initial center of mass of our chain is

$$y_{cm} = \frac{0\left(\frac{m}{2}\right) + \left(-\frac{L}{4}\right)\left(\frac{m}{2}\right)}{m}$$

$$y_{cm} = -\frac{L}{8}$$

The chain center of mass only falls distance $\frac{3L}{8}$…not $\frac{L}{2}$.
As such we expect the chain to have less speed than a ball falling more distance.

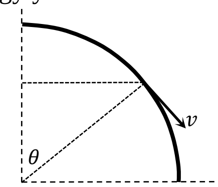
We can go even further:
A ball released from rest which falls distance $\frac{3L}{8}$ should acquire speed

$$v = \sqrt{2g\left(\frac{3L}{8}\right)} = \sqrt{\frac{3}{4}gL}$$

Notice this result is the same as part a!
Also notice: the normal force does no work on the chain as it falls.
That force is perpendicular to the displacement of each chain link.

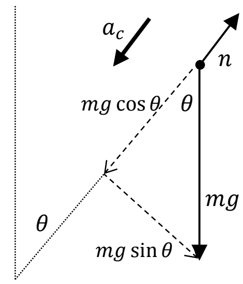
8.36

a) For circles it often makes sense to choose a reference point at the center. If you do this, we are free to use the equation $x^2 + y^2 = R^2$ if needed. This makes the ground the reference point. The normal force does no work as it is perpendicular to displacement. No friction is present. Conservation of energy yields



$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$$
$$mgR = \frac{1}{2}mv^2 + mgR\cos\theta$$
$$v = \sqrt{2Rg(1-\cos\theta)}$$

If you have trouble with the geometry, revisit the pendulum problem (**8.16**).
An FBD is shown at right. The force equation is



$$mg\cos\theta - n = ma_c = m\frac{v^2}{R}$$

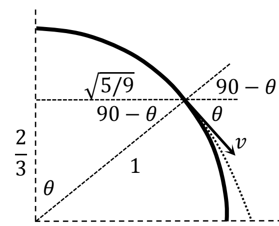
When the block loses contact we know $n = 0$ This yields

$$g\cos\theta = \frac{v^2}{R}$$

Comparing to the energy problem gives

$$\theta = \cos^{-1}\left(\frac{2}{3}\right) \approx 48.2°$$

b) We know $\theta = \cos^{-1}\left(\frac{2}{3}\right) \approx 48.2°$ (or $\cos\theta = \frac{2}{3}$). Using $g = 9.8\,\frac{\text{m}}{\text{s}^2}$ this we find



$$v = \sqrt{2Rg(1-\cos\theta)} = \sqrt{\frac{2}{3}Rg} = 2.5560\,\frac{\text{m}}{\text{s}}$$

The height at which the ball leaves the dome is $\frac{2}{3}R = \frac{2}{3}$. Pythagorean theorem gives the other side of the triangle and gives $\sin\theta = \sqrt{\frac{5}{9}} \approx 0.74535$. I'm keeping extra sig figs in case a subtraction occurs.

I choose to ignore units to reduce clutter.

$$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y$$
$$v_{fy} = -4.086$$

Used negative root because moving downwards at impact.
The time to fall can now be found using

$$v_{fy} = v_{iy} + a_y t$$
$$t = \frac{v_{fy} - v_{iy}}{a_y}$$
$$t = 0.2225$$

x_i	$\sqrt{\frac{5}{9}} \approx 0.7454$	y_i	$\frac{2}{3}$
x_f	?	y_f	0
v_{ix}	$v\cos\theta = 1.7040$	v_{iy}	$-v\sin\theta = -1.9051$
v_{fx}	$v\cos\theta = 1.7040$	v_{fy}	?
a_x	0	a_y	$-g$
t	?		

Therefore the final position is

$$x_f = x_i + v\cos\theta\, t$$
$$x_f = 1.1246$$

Therefore the ball lands distance $0.1246R \approx \frac{1}{8}R = 12.5$ cm from the edge of the dome.

8.37 I used $g = 10\frac{\text{m}}{\text{s}^2}$ for this problem and rounded to 2 sig figs.

a) At the bottom the force equation is $T_{max} - mg = m\frac{v^2}{r}$. This gives $v_B = \sqrt{r\left(\frac{T_{max}}{m} - g\right)} \approx 28\frac{\text{m}}{\text{s}}$.

b) Use energy to get the speed at the top.

$$\frac{1}{2}mv_B^2 = \frac{1}{2}mv_T^2 + 2mrg$$
$$v_T = \sqrt{v_B^2 - 4rg}$$
$$v_T = \sqrt{r\left(\frac{T_{max}}{m} - g\right) - 4rg}$$
$$v_T = \sqrt{\frac{rT_{max}}{m} - 5rg} \approx 2\underline{6}.8\frac{\text{m}}{\text{s}}$$

c) At the top the force equation is

$$T_{top} + mg = m\frac{v_{top}^2}{r}$$
$$T_{top} + mg = \frac{m}{r}\left(\frac{rT_{max}}{m} - 5rg\right)$$
$$T_{top} + mg = T_{max} - 5mg$$

$$T_{top} = T_{max} - 6mg$$
$$T_{top} = 50mg - 6mg$$
$$T_{top} = 44mg$$

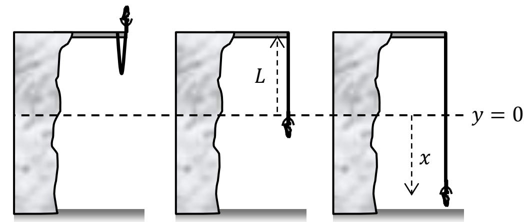
The ratio is thus

$$\boldsymbol{\frac{T_{top}}{T_{bot}} = 0.88}$$

In other words, the tension at the top is 16% of the tension at the bottom.

d) It is possible if we change the mass to some new value m'!
At the top we want the minimum speed to maintain circular motion.
This requires $T = 0$ and yields $v_T = \sqrt{rg}$.
Using energy as in part b, the speed at the bottom is $v_B = \sqrt{v_T^2 + 4rg} = \sqrt{5rg}$.
While on the verge of breaking the string at the bottom the FBD gives $T_{max} - m'g = m'\frac{v^2}{r}$.
Plugging in our speed at the bottom gives $T_{max} = 6m'g$.
It is possible to design such a system by choosing the appropriate mass (let $6m' > 50m$ or $m' \approx 8.4m$).

8.38



a) Speed is $v = \sqrt{2gL}$ because the fall to the unstretched position is still freefall (if the cord is massless).

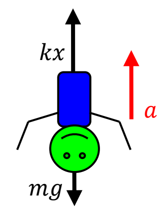
b) I chose to set the dotted line (unstretched position) as $y = 0$.
I compared the initial stage to max stretch stage so both the initial and final kinetic energies would be zero. Here x is *distance stretched* by the spring. The final vertical *position* is $y = -x$.

$$\frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2 + W_{friction} = \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2$$

$$mgL = -mgx + \frac{1}{2}kx^2$$

Doing the quadratic formula gives $x = \frac{mg}{k}\left(1 + \sqrt{1 + 2\frac{kL}{mg}}\right)$. Remember x is distance not position. As a result, the positive root makes sense in your quadratic formula.

Max Stretch FBD



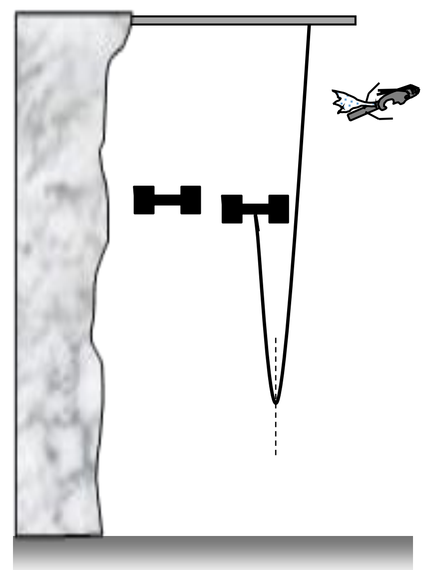
c) The distance below the bridge for the entire fall is $d = L + \frac{mg}{k}\left(1 + \sqrt{1 + 2\frac{kL}{mg}}\right)$.

d) One finds $= 10\text{ m} \approx 33\text{ ft}$ $d = 25\text{ m} \approx 82\text{ ft}$.

e) Consult the FBD at right. The acceleration is $a = \frac{kx}{m} - g = 40 \approx 4g$. That jumper might go unconscious…

f) If the system is swinging there must be an increase in the cord tension because the acceleration upwards would be greater by an amount equal to centripetal acceleration (times mass). The only way for the cord to produce more tension is for it to have greater elongation.

8.39 The mass tied to the chain will impact first. Consider splitting the chain where it bends at the bottom. The left portion is moving downwards at essentially the same rate as the mass. The portion on the right is not moving significantly. As parts of the chain transition from the left side to the right, the right side must pull upwards on it to change the motion from moving down to approximately at rest. Therefore the chain has tension in it. This means the mass attached to the chain has a net downwards force greater than its weight. The mass with the chain should impact first. The mass with the chain has average acceleration magnitude greater than g!



Try a web search for "chain drop experiment".

8.40

a) The student forgot to account for centripetal acceleration.
Also, the angle for circles is usually drawn to the vertical while in this circle it is drawn to the horizontal. As such, this student should've used $\sin\theta$ instead of $\cos\theta$.

b) In this problem $W_{non-con} \neq 0$.
The equation $v = \sqrt{2g|\Delta h|}$ is only valid if $W_{non-con} = 0$.
Too bad, eh?

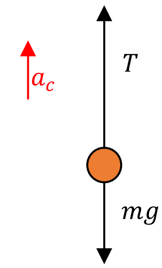
8.41 Tension is greatest at the bottom of the swing so that is where it is likely to break (exceed load limit of 800 N). First show the speed at the bottom of the swing is given by

$$mgh_i = \frac{1}{2}mv^2$$

$$gL(1-\cos\theta) = \frac{1}{2}v^2$$

$$v = \sqrt{2gL(1-\cos\theta)}$$

An FBD at the bottom of the swing gives rise to the force equation



$$T - mg = m\frac{v^2}{L}$$

$$T - mg = 2mg(1-\cos\theta)$$

Notice string length drops out!!!

$$T = 3mg - 2mg\cos\theta$$

$$\frac{T}{mg} = 3 - 2\cos\theta$$

$$\cos\theta = \frac{3}{2} - \frac{T}{2mg}$$

Here T is the max tension of the string ($T_{max} = 800\text{ N} = \frac{4}{3}mg$) if it is about to break.

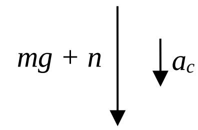
$$\theta = \cos^{-1}\left(\frac{5}{6}\right) = 33.6°$$

8.42 initial speed of the block is $v_0 = \sqrt{3Rg}$ and it will separate from the track when $\theta = 30°$.

a) I assumed the initial position had zero height. Comparing the starting position to the top of the hill with energy one finds

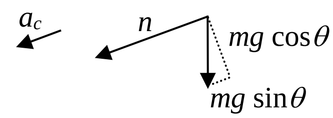
$$\frac{1}{2}mv_0^2 = mgR + \frac{1}{2}mv_{top}^2$$

From an FBD at the top of the hill we expect $mg + n = ma_c$. Since it is about to lose contact we assume $n \approx 0$ which gives $a_c = g$. From this we know $\frac{mv_{top}^2}{R} = mg$ or $v_{top}^2 = Rg$.



Some algebra gives the result $v_0 = \sqrt{3Rg}$.

b) We can now do an FBD for a random angle on the bigger circular portion of the track. **Notice the radius is now $2R$. Now we must use $a_c = \frac{v^2}{2R}$.**



Note: I will assume the block loses contact and falls off the track at some point. Notice, if the block loses contact at angle θ the height of that position is necessarily $h_f = 2R\sin\theta$. If the calculated gives a nonsense answer for θ, we can assume the block makes it to the end of the track.

If the block loses contact we again have $n \approx 0$ which this time gives (from the FBD and force equation)

$$ma_c = mg\sin\theta$$
$$\frac{v_f^2}{(2R)} = g\sin\theta$$
$$v_f^2 = 2Rg\sin\theta$$

The energy equation (comparing *initial* position to *final* position…ignoring the first hill) gives

$$\frac{1}{2}mv_0^2 = mgh_f + \frac{1}{2}mv_f^2$$
$$\frac{1}{2}m(3Rg) = mg(2R\sin\theta) + \frac{1}{2}m(2Rg\sin\theta)$$
$$3 = 4\sin\theta + 2\sin\theta$$
$$\theta = \sin^{-1}\left(\frac{3}{6}\right) = 30°$$

Notice, for this case, $h_f = 2R\sin(30°) = R$.

c) If the second radius was $3R$ instead of $2R$ the equations become

$$h_f = 3R\sin\theta$$
$$v_f^2 = 3Rg\sin\theta$$
$$\frac{1}{2}m(3Rg) = mg(3R\sin\theta) + \frac{1}{2}m(3Rg\sin\theta)$$
$$3 = 6\sin\theta + 3\sin\theta$$
$$\theta = \sin^{-1}\left(\frac{3}{9}\right) \approx 19.5°$$

Notice, for this case, $h_f = 3R\sin(19.5°) = R$ once again! Far out.

Makes you wonder if the final height is always R. Well…it is.

Replace $3R$ by r in the previous equation and you should find $\frac{3}{2}R = \frac{3}{2}r\sin\theta = \frac{3}{2}h_f$. This shows $h_f = R$.